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Correlation Of Input And Resulting Audit Measures For An Automotive Axle Shim Selection Process Using A Monte Carlo Simulation Assessment Of Accuracy

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CORRELATION OF INPUT AND RESULTING AUDIT MEASURES FOR AN AUTOMOTIVE AXLE SHIM SELECTION PROCESS USING A MONTE CARLO SIMULATION ASSESSMENT OF ACCURACY

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ABSTRACT

The demands on the automotive industry for increased reliability and reduced noise have a direct effect on automotive axle requirements. This demand translates to increased precision in automotive axle manufacturing, including axle assembly where accurate positioning of hypoid gears and setting of proper bearing axial load is the most challenging process. To achieve the accuracy required, the axle assembly often includes select fit shims to control gear position and bearing preload force. The shim selection process integrates a measurement system into the assembly process that includes; in-process measurements of components and subassemblies as inputs, and audit measurements of each assembly to confirm gear position by backlash and bearing preload by torque to rotate as outputs. Understanding the correlation of in-process measurements to audit measurements is an essential part of optimizing the shim selection process.

The purpose of this research was to define and assess a method to correlate input measurements as independent variables to audit measurements as dependent variables in an axle assembly system. This correlational study developed and assessed an axle shim selection process model as a predictor of variance in the dependent variables of backlash and rotational torque. To account for errors affecting shim selection the measurement uncertainty framework was used. The study included three steps. The first step developed an uncertainty model of an existing axle assembly measurement system using the standard uncertainty propagation method. The second step evaluated the ability of the model to simulate the production process and predict process capability using a Monte Carlo Method (MCM) simulation. The third step applied the model to assess effects of a measurement error for the axle cover in-process measurement.

The results of this study suggest that an uncertainty model can correlate input and output measurements in the shim selection process. Through regression analysis of reworked axles, a statistically significant linear correlation between shim thickness change and the dependent variables of backlash and bearing torque to rotate was identified. The coefficients from the regression analysis combined with the measurement uncertainty components were included in a predictive MCM simulation. Results from the simulation were compared to production data to evaluate the effectiveness of the model at predicting system performance. The model simulation did predict system first time acceptance through the shim selection process, MCM results were within 0.8% of backlash and 0.2% of bearing torque to rotate when compared to sample production data. Though there was a statistical difference in the prediction of backlash, the effect was not practically significant. The study identified that factors not directly associated with the assembly measurement process are a significant contributor, repeatability GR&R studies alone were insufficient in explaining the overall process error. Hypothesis testing suggests that the application of the measurement uncertainty framework that includes the effects of statistical and non-statistical contributors to process error can predict automotive axle shim selection process capability.

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CHAPTER 1

THE PROBLEM AND ITS SETTING

In automotive axles, hypoid gears have been the dominant transmission gearing since their introduction in 1926 (Coy, Townsend, & Zaretsky, 1985). Stewart & Wildhaber(1926) identified hypoid gearing benefits as high power density, reduced noise, and extended life. Stadtfeld (2011a) refers to hypoid gears as the paragon of gearing in the transmission of power at angles, commonly ninety-degrees as required in many automotive axle applications. The assembly process in automotive axles includes two requirements. First, accurate positioning of hypoid gears to maintain gear meshing performance (Wang, Fang, & Li, 2014), secondly, maintaining adequate bearing preload (Timken, 2011). To achieve the required bearing preload and gear position accuracy, inserting select fit shims or spacers are a common practice. Select fit assembly typically includes a complex measurement system. Such a measurement system combines component and subassembly measurement, with audit measurement that assesses the position of the gears by measuring gear backlash (Stadtfeld, 2014) and bearing preload by measuring torque to rotate. To determine the shim-selection process suitability, a measurement system analysis (MSA) method is required.

In automotive manufacturing, the Automotive Industry Action Group (AIAG, 2010) publication Measurement Systems Analysis Reference Manual, commonly referred to as MSA, is widely used in the specification and assessment of measurement systems. The AIAG measurement process analysis includes assessing the value of measurement process. An over-specified measurement process adds unnecessary equipment acquisition and operational cost. An underspecified shim-selection measurement process produces inaccuracy and resulting incorrect shim selection that requires rework of assembled axles. Shewhart (1939) discussed the need for measurement accuracy and precision and the importance of acquiring enough data to support decisions. To achieve this balance requires what Dietrich and Schulze (2011) define as an appropriate measurement system that supports the correct manufacturing process.

Assessing measurement system capability is the subject of several standards that provide guidelines, describe procedures, and establish criteria (Dietrich & Schulze, 2011). Montgomery and Runger (1993) identified that the purpose of measurement system capability analysis is to understand and quantify the variability present in the measurement process. The traditional approach to measurement system analysis is Gauge Repeatability and Reproducibility (GR&R) studies. Various assessment methods are applied; AIAG MSA(2010) guidelines include Average and Range and ANOVA Methods, Wheeler (2009) proposes a method he describes as Honest Gauge R&R, Ingram & Taylor (1998) apply and Analysis of Means method. In all methods, the outcome of the measurement system capability analysis is an estimate of the standard deviation of the measurement applied to specification or process limits. These methods are not directly applicable to the shim selection process that applies input measurements to control the output as determined by audit measurements.

In automotive axle manufacturing, controlling the individual components at the precision required to achieve axle assembly gear position and bearing preload requirements is not practical. In lieu of controlling individual components, automotive axle assemblies include select fit shims to achieve these requirements as shown in Figure 1. Select fit axle shims are nominally 3 mm thick and produced in 0.025 mm thickness increments. To determine the thickness of these shims, each individual axle includes component measurements as part of the assembly process.

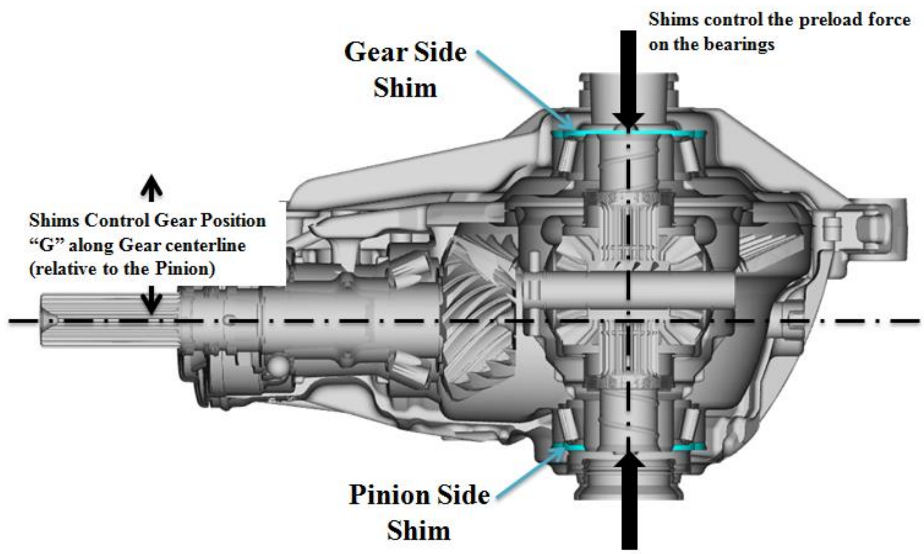

Figure 1. Axle assembly with select fit Pinion Side and Gear Side case shims

This shim selection process includes two measurement types, in-process measurements and 100% verification measurements called the audit measurements. The results of the in-process component and sub-assembly measurements are used in selecting the shim size required to achieve backlash and bearing preload on individual axle assemblies. The in-process measurements for a sample axle are shown in Figure 2. Most of the in-process measurements are linear dimensions determined by measuring the components during the assembly process. The exception is the measurement δJ. This measurement is a deviation from the ideal mounting position of the ring gear "G" and is determined uniquely for each gearset during the gear manufacturing process. To reduce variation in the shim selection process the first shim selected is measured and included in the second selected shim calculation. In this example, the Pinion Side Shim thickness (PS_{Meas}) is measured and included as part of the Gear Side Shim thickness calculation.

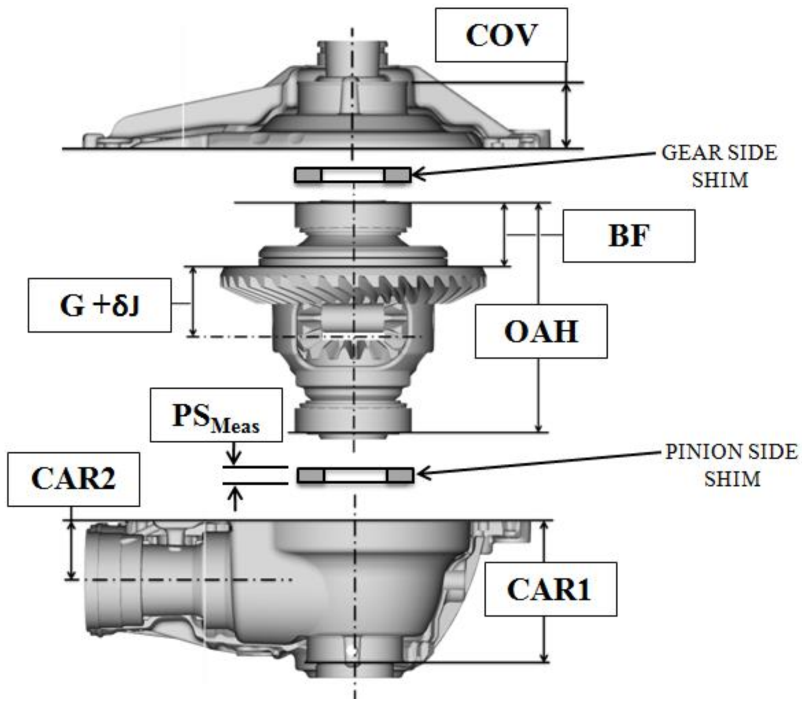

Figure 2. Axle assembly in-process measurements for case shim selection

These seven in-process measurements are combined to determine the select fit shims as shown in Equations 1 and 2. In addition to the measured input variables, included in each of the shim selection equations are bias compensations referred to as "production offsets" that are used to provide an additional control of the process. During axle production, the verification audit measurements are monitored to validate that the process is centered. Similar to CNC machines, if the process is not centered a method of slightly modifying each calculated shim thickness is to add or subtract a constant value referred to as an offset. The offsets are typically less than 0.050 mm and typically remain constant throughout a day's production.

$$Pinion\ Side\ Shim\ thickness = CAR1 - CAR2 + (G + \delta J) - (OAH - BF) + OFFSET_{PS} \quad (1)$$

$$Gear\ Side\ Shim\ thickness = CAR1 + COV - OAH - PS_{Meas} + OFFSET_{GS} \quad (2)$$

Once selected, the shims are inserted and the axle is assembled. As part of the assembly process, each axle is measured to validate that gear position and bearing preload are correct. The position of the gear and the bearing preload force cannot be directly measured. Two indirect measurements are used; gear position by gear backlash, and bearing preload by bearing rotational torque. These measurements, referred to as process audit measurements, are influenced by uncertainties in the measurement process. These uncertainties include measurement error and uncertainties associated with variables that are not measured, such as the variation in Pinion position in the assembly. If the assembly audit measurements are outside of product tolerance limits, the axle is rejected and returned to the process for rework and reassembly with new shims. An assembly system for an automotive shimmed axle is shown in Figure 3. Accepted parts are OK and reject parts are not OK (NOK).

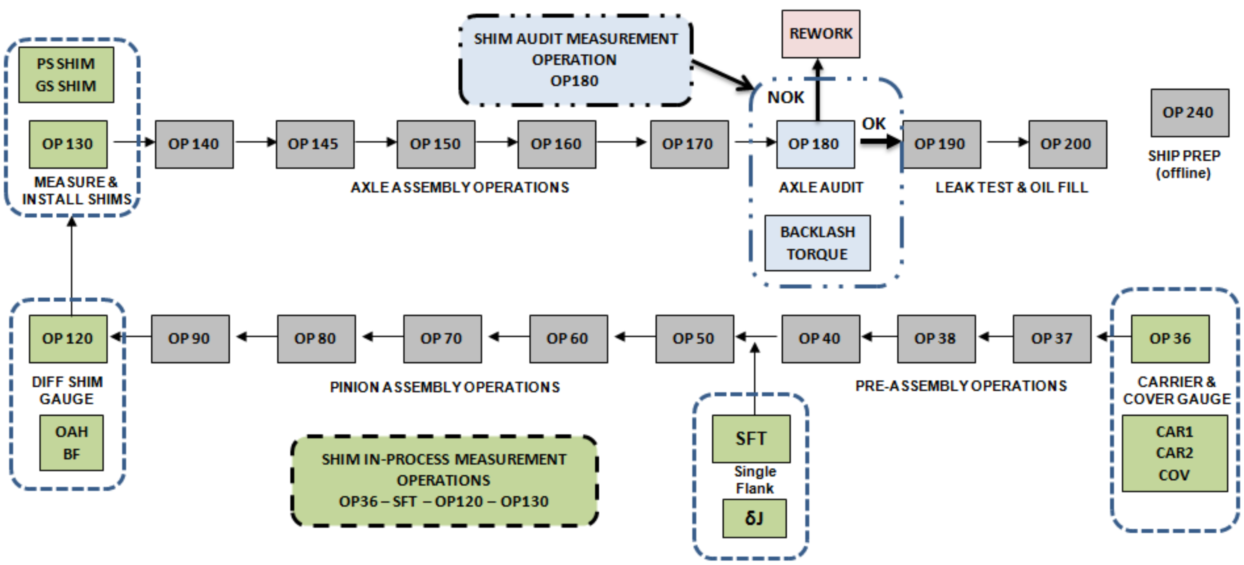

Figure 3. Axle assembly and shim-selection measurement process block diagram

In automotive axle assembly, the measurement system is an integral part of the assembly process. The in-process measurements are inputs to the shim selection process. The audit measurements compared to acceptance limits are the process outputs. In this manufacturing environment, a conventional measurement system analysis criterion is not directly applicable. AIAG recognized that the MSA is not a "compendium of analysis for all measurement systems" (AIAG, 2010). The application of published MSA methods to axle shim selection require expanded analysis methods to support: (a) the determination of what individual measurement accuracy is required for process control, (b) the influences of variables not measured, and (c) a priori performance assessment. A method that correlates direct input measurements to indirect output measurements can provide this expanded analysis method.

Uncertainty Analysis in Measurement Systems

A method for assessing and reporting the accuracy of a measurement is uncertainty analysis that Coleman and Steele (2009) describe as the degree of goodness of a measurement. Uncertainty analysis is a statistical approach to document the confidence of a measurement (Hughes & Hase, 2010). It provides a method to include other contributors to measurement error beyond the typical repeatability study (Dietrich, 2014). There exists a distinction between measurement uncertainty and measurement capability. AIAG MSA Reference Manual (2010) draws a contrast between measurement uncertainty, which is an assessment of the confidence of a measurement, and measurement system analysis, which is a system for understanding and improving a measurement process.

Uncertainty may be expressed as standard uncertainty u, which is synonymous with standard deviation, and u^2which is synonymous with variance (Dietrich & Schulze, 2011). This is the method used in the more recent International Standards Organization (ISO) Standard 22514-7 (2012) that includes measurement uncertainty analysis as part of assessing the measurement system capability. The effects of measurement uncertainty on process variation can be expressed by considering standard uncertainty as standard deviation in the MSA (AIAG,

2010) relationship $\sigma^2_{observed} = \sigma^2_{process} + \sigma^2_{measurement}$. Applying the substitution to the ISO (2012) expression for observed to the actual process capability index (C_p), as $Cp_{Act} = Cp_{Obs}\sqrt{1 + \left(\frac{\sigma_{measurement}}{\sigma_{Act}}\right)^2}$ the effects of measurement uncertainty may be quantified. This is illustrated in Figure 4 where the effects of measurement uncertainty add to the actual process variation. The actual process capability must outperform the observed capability based on the measurement uncertainty.

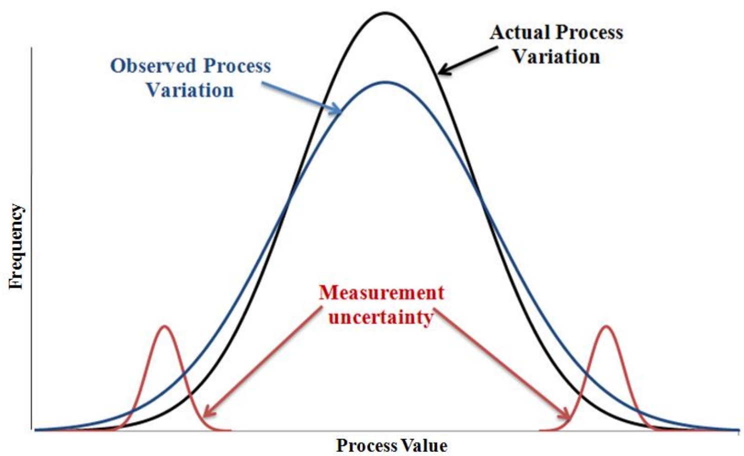

Figure 4. Influence of measurement uncertainty on observed process capability(AIAG, 2010)

In reference to uncertainty in measurement, all of the sources listed above refer to the Guide to the Expression of Uncertainty in Measurement (GUM) (Joint Committee for Guides in Metrology, 2008a) as the source for evaluation and expression of uncertainty. Taylor and Kuyatt (1994) National Institute of Standards and Technology (NIST) provide a Technical Note as guidance for evaluating and expressing the uncertainty, noting that the NIST is intended as a more condensed document consistent with the GUM. The GUM provides a recognized method to express two types of uncertainty, one based on statistical analysis of observations, and one by means other than statistical analysis of observations.

The topics of measurement, measurement systems, and uncertainty have been the subject of previous research. Stamm (2013) researched measurement system analysis techniques comparing ANOVA, MSA, and Wheeler's Honest Gauge R&R approaches. Patki (2005) researched methods to improve the techniques of existing MSA methods. Fleming (1988) discussed the issues with dimensional tolerances and uncertainty relative to assembly processes. The previous research does not address the measurement requirements of the axle shim-selection assembly processes.

United States automotive manufacturing organizations often look at the AIAG MSA as the authoritative reference for acceptance criteria of a measurement system. MSA(2010, p. 62) notes that their approach assesses a measurement system as a process control tool. Summarizing the AIAG, ISO, and German Automotive Association (VDA), Dietrich (2014) similarly states that the measurement process must be compared to the specification or manufacturing process limits. These processes typically apply assessment techniques like the Automotive Industry Action Group (AIAG) Gauge Repeatability and Reproducibility (GR&R) studies designed for assessing the measuring process capability (2010). For the axle shim-selection process, these approaches do not provide a method to correlate input and audit measurements or predict the performance of the measurement and assembly process. A gap exists between the published and accepted measurement system analysis methods and the requirements of an axle shim-selection system analysis.

This study attempted to address this gap by employing measurement uncertainty methods to the axle shim-selection process. Uncertainty methods are typically applied to report measurement confidence intervals; however, in this study, they were applied as a method to predict process outcomes. The basis of this study is that measurement uncertainty methods can be applied as a measurement system analysis tool for axle shim-selection. To test this theory, a production shim selection measurement system was modeled using uncertainty methods including correlation of constituent input measurements with audit measurements. The accuracy of the model prediction was assessed using a Monte Carlo simulation to compare model data to production data. The study aimed to determine the application of this approach to predict future axle shim selection systems and establish acceptance criteria.

This study applies the technology management principle of using technology to solve problems and improve an organizations efficiency. By transferring existing technology to the shim selection process the study provides a method to improve the efficiency of an established practice. The development of an assessment method for shim selection provides a tool for management that permits process improvements while potentially providing an economic benefit by reducing cost and improving process capability.

Statement of the Problem

A gap exists between the published and accepted measurement system analysis methods and the requirements of an axle shim-selection system analysis, therefore there is a need to develop a prediction method to correlate input and audit measures of an axle shim-selection process.

Primary Research Question

Can the application of uncertainty principles in measurement provide a method to correlate input variables and predict performance of the shim-selection process output variables of backlash and torque to rotate?

Specific Research Questions (RQ)

RQ 1: How can measurement uncertainty methods be applied to model the axle shim-selection measurement process?

RQ 2: Can a measurement system uncertainty model be used to predict the backlash and torque-to-rotate capability of a shim-selection measurement system?

RQ 3: Can a measurement system uncertainty model be used to determine the acceptance limits for an individual in-process shim-selection measurement apparatus?

Research Question 1; how can measurement uncertainty methods be applied to model the axle shim-selection measurement process? The first specific research question involves the development of a model to predict the performance of the measurement system in the axle assembly process. The development of the model results from the application of measurement uncertainty techniques as outlined in literature to a specific axle shim-selection manufacturing process.

Research Question 2; can a measurement system uncertainty model be used to predict the backlash and torque-to-rotate capability of a shim-selection measurement system? The second specific research question involves validation of the model. It is answered by comparing actual axle assembly shim-selection system production data to the data from a Monte Carlo simulation. A data set is developed using Monte Carlo simulation of measurement true values as input to the model developed in Research Question 1. This comparison is made through the following hypotheses:

H_{O1}: There is no significant difference between the Means of the uncertainty prediction model and actual test data in Backlash Audit.

H_{A1}: There is significant difference between the Means of the uncertainty prediction model and actual test data in Backlash Audit.

H_{O2}: There is no significant difference between the Variance of the uncertainty prediction model and actual test data in Backlash Audit.

H_{A2}: There is significant difference between the Variance of the uncertainty prediction model and actual test data in Backlash Audit.

H_{O3}: There is no significant difference between the Means of the uncertainty prediction model and actual test data in Audit Total Torque to Rotate.

H_{A3}: There is significant difference between the Means of the uncertainty prediction model and actual test data in Total Torque to Rotate.

H_{O4}: There is no significant difference between the Variance of the uncertainty prediction model and actual test data in Total Torque to Rotate.

H_{A4}: There is significant difference between the Variance of the uncertainty prediction model and actual test data in Total Torque to Rotate.

Research Question 3; can a measurement system uncertainty model be used to determine the acceptance limits for an individual in-process shim-selection measurement apparatus? The third research question involves varying measurement capability of a selected in-process measurement and assessing the impact on process results using the simulation model developed in Research Questions 1. Two model simulation runs were used to compare results with Cover bearing bore depth (COV) at AIAG MSA 10% GR&R and 50% GR&R with the tolerance range as upper and lower limits using the following hypotheses.

H_{O5}: There is no significant difference between the Means of the uncertainty prediction model Audit Backlash with COV measurement capability at 10% and 50%.

H_{A5}: There is significant difference between the Means of the uncertainty prediction model Audit Backlash with COV measurement capability at 10% and 50%.

H_{O6}: There is no significant difference between the Variance of the uncertainty prediction model and Audit Backlash with COV measurement capability at 10%

and 50%.

H_{A6}: There is significant difference between the Variance of the uncertainty prediction model Audit Backlash with COV measurement capability at 10% and 50%.

H_{O7}: There is no significant difference between the Means of the uncertainty prediction model Audit Torque-to-Rotate with COV measurement capability at 10% and 50%.

H_{A7}: There is significant difference between the Means of the uncertainty prediction model Audit Torque-to-Rotate with COV measurement capability at 10% and 50%.

H_{O8}: There is no significant difference between the Variance of the uncertainty prediction model Audit Torque-to-Rotate with COV measurement capability at 10% and 50%.

H_{A8}: There is significant difference between the Variance of the uncertainty prediction model and Audit Torque-to-Rotate with COV measurement capability at 10% and 50%.

Purpose of the Study

The purpose of this research was to define and assess a method to predict the performance of an axle shim-selection measurement system. The study used three sequential steps to achieve this. First, the study modeled an existing production shim selection system to characterize the relationship between independent variables and the dependent variables of backlash and torque to rotate using uncertainty techniques. Second, a Monte Carlo simulation compared the model prediction to the production measurement system. Third, the study assessed the effects of measurement capability for a specific independent variable on the performance of the system.

Need for the Research

There is a need for a method to correlate the input measurements to the audit measurements and predict the capability of the shim selection process. Applying AIAG MSA criteria, that MSA(2010) describes as applicable to measurement systems for process control, does not address this specific requirement. The International Standards Organization ISO-22514-7 (ISO, 2012) applies uncertainty analysis but the capability of the measurement system is assessed by applying upper and lower limits of the measurand, not suitable for the shim selection process. Other measurement process qualification standards reviewed by Dietrich and Schulze (2011) also rely on process variation or drawing tolerance limits as the system assessment criteria. Therefore, as stated above, a need exists to develop a method to analyze the axle shim-selection measurement system thereby allowing process improvements and reducing cost for axle manufacturers. This method needs to account for individual test article variation and input of known sources of uncertainty in predicting the outcome of backlash and rotational torque.

Delimitations and Clarifications

The basis for measurement apparatus repeatability typically includes an assessment of the variance or standard deviation of the measurement. In most instances, this study uses the Average and Range method for estimating the measurement standard deviation from the average range $\bar{R}$ using the d_2^* estimator, $\sigma_{est} = \bar{R}/d_2^*$. The ANOVA method is applied in instances where more data is available or where there is a desire to identify interactions (AIAG, 2002).

An element of uncertainty analysis as outlined by the GUM is an evaluation of uncertainty by means other than statistical analysis of a series of observations, which is deemed Type B evaluation of uncertainty. This study includes estimates of uncertainty by non-statistical means. The application of Type B in this study includes known variables, such as feature non-homogeneity, that are difficult to measure and analyze statistically. One of the elements of this research is to provide a method to include such considerations in a measurement process, and assess the effectiveness in sample data.

Two terms are common in uncertainty analysis, standard uncertainty u_i comparable to standard deviation, and expanded uncertainty U_i. The expanded uncertainty is applied to represent the statistical range of uncertainty of a measurement and is calculated based on a coverage factor k, $U_i = ku_i$. The coverage factor is not applicable to this study, uncertainty as applied is the standard uncertainty.

Assumptions of the Study

It is an assumption of this study that the part variability associated with rolling element bearings is a contributor to the overall measurement system uncertainty. It is not within the scope of this study to characterize this variation. Factors that vary during the assembly measurement process, such as variation in lubrication conditions and the run-in time, result in variation in rolling torque (Timken, 2011, p. 180).

Data collected for this study is by manufacturing facility SCADA systems from automatic gauges, recorded and stored electronically. Control networks are discussed by Moyne and Tilbury (2007) for manufacturing control and networking that enables automatic inspection, and traceability through part identification. Individual component barcodes provides the capability to transfer component specific measurement data to the assembly process. An assumption of this study is that this process provides accurate recording and data transfer. The measurement data acquisition is automatic without operator influence and without a requirement for human discretion or recording of a measurement. The manufacturing facility performs regular audits to validate the data recording accuracy and reliability.

In some instances, the measured components are positioned manually into an automatic gauge. In selected instances where the test article is manually loaded, it is an assumption of this study that the effect of manual activity is not a factor in the measured results; in other instances, a reproducibility study is applied to assess operator influence. The specific assumption is clarified and documented for each measurement.

Several of the techniques of uncertainty analysis include a requirement that the process is normally distributed and that measurements are independent. This assumption was validated through analysis of data or clarified as an assumption during the data analysis process for each variable.

Terminology

AIAG (Automotive Industry Action Group) - Organization founded in 1982 by the three largest North American automotive manufacturers – Chrysler, Ford, and General Motors. AIAG is a not-for-profit association that now includes automotive manufacturers and suppliers that provides educational programs and standards publication (AIAG, 2015).

Accuracy – The agreement of a measured value with an accepted reference value (AIAG, 2010).

Audit – For the purposes of this study, the measurement of a feature after a manufacturing process that is used to assess the acceptance of the product to a limit.

Average Range Method – For the purposes of this study, average range method is a statistical estimate of standard deviation based on a range of measurements using the d_2^* parameter (AIAG, 2010).

Bias – Deviation of a measurement or average of measurements form the true value of the characteristic (Dietrich & Schulze, 2011).

BIPM (Bureau International des Poids et Measures) – The International Bureau of Weights and Measures, an “intergovernmental organization established by the Metre Convention, through which Member States act together on matters related to measurement science and measurement standards” (BIPM, 2013).

Carrier - The carrier in an axle assembly is a reference to the cast housing machined to position the hypoid gears and for load reaction in the vehicle.

Combined standard uncertainty– The “standard uncertainty of the result of a measurement when that result is obtained from the values of a number of other quantities” (JCGM, 2008a).

Differential Shims - Selectable spacers located between the Carrier and the bearing to position the ring gear in a shimmed style axle, also referred to as case shims.

DTR (Differential Torque to Rotate) – The torque required to rotate the differential assembly in the axle resulting from axial preload on the differential bearings.

Error– For purposes of this study error refers to the difference between a measurand observed value and the true value.

Expanded uncertainty (U) – The “quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand” (JCGM, 2008a).

First Time Acceptance (FTA) – An assembly completing the process and is accepted the first time without rework, synonymous with First Time Quality (FTQ).

GR&R (Gauge Repeatability and Reproducibility) – Estimate of the combined variation of repeatability and reproducibility of a measurement system (AIAG, 2010, p. 215).

GUM – A reference to the BIPM publication *Evaluation of measurement data - Guide to the expression of uncertainty in measurement*.

Hypoid Gears - High efficiency gears in terms of combined power transmission and efficiency, Dr. Hermann Stadtfeld (2011a) describes them as the paragon of gearing. In automotive applications, the gears are often lapped as matched pairs that require accuracy in positioning in the final assembly.

ISO (International Organization for Standards) – A global federation of national standards bodies that prepare and publish standards through technical committees (ISO, 2012).

Measurand – The particular item that is being measured or subject to measurement under specified conditions. (AIAG, 2002)

Measurement Uncertainty – "non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used" (JCGM, 2008b).

Measurement System Analysis (MSA) – Quantification of measurement error by analyzing multiple sources of variation in a process including variation from the; measurement system, evaluators, and measured articles (Kazemi, Haleh, Haijpour, & Rahmati, 2010, p. 25).

Monte Carlo Method (MCM) – "[A] method for the propagation of distributions by performing random sampling from probability distributions" (JCGM, 2008c).

Pinion - The Pinion is the smaller gear in a hypoid gear-set connected to the driveshaft in a driving axle.

Precision – Describes the variation of repeated measurements over the range of measurement, synonymous with repeatability (AIAG, 2010).

PTR (Pinion Torque to Rotate) - The torque required to rotate the pinion assembly in the axle resulting from axial preload on the pinion bearings

Ring Gear - The Ring Gear is the larger gear in a hypoid gear-set, referred to synonymously as; ring, gear, and crown gear.

Single Flank Test (SFT) – A gear processing measurement that characterizes the gear form on the concave and convex flanks separately. Used in the manufacture of hypoid gears to measure transmission error (Stadtfeld, 2011b).

SCADA (Supervisory Control and Data Acquisition) - A computerized system that is capable of gathering and processing data and applying operational controls over distances where centralized data acquisition and control are critical to system operation (Stouffer, Falco, & Scarfone, 2011).

Standard uncertainty (u) – An "uncertainty of the result of a measurement expressed as standard deviation" (JCGM, 2008a).

True Value – The actual measure of the part, though unknowable it is the target of the measurement process (AIAG, 2010, p. 45).

TTR (Total Torque to Rotate) – The torque required to rotate the assembled axle resulting from axial preload on the pinion and differential bearings. TTR is related to PTR and DTR by the theoretical relationship, $TTR = PTR + TTR/Gear\ Ratio$.

Type A Uncertainty – "[A] method of evaluation of uncertainty by the statistical analysis of a series of observations." (JCGM, 2008a, p. 3)

Type B Uncertainty – "[A] method of evaluation of uncertainty by means other than the statistical analysis of a series of observations." (JCGM, 2008a, p. 3)

Uncertainty Analysis – Analysis of the experimental uncertainties in measurements and in simulation results (Coleman & Steele, 2009, p. 5).

CHAPTER 2

REVIEW OF LITERATURE

Mass production manufacturing transitioned in 1924, Shewhart(1939, pp. 4-5) identifies the growth in standardization and the acceptance of probability and statistics in the sciences, as motivation for the adoption of the quality control chart, the starting point of statistical process control (SPC). To establish whether a process is under statistical process control requires, as Shewhart(1939) describes, measurements of one kind or another i.e., a measurement system. As part of that process, Shewhart(1939) discusses the need for accuracy and precision in measurement. He concluded that it was impossible to specify authoritatively a definitive meaning for either.

This chapter provides a literature review on the application of statistics and uncertainty in measurement systems. Previous research provides the background on methods for expressing and applying uncertainty techniques. Included in this chapter review is literature on statistics as background in measurement and uncertainty analysis applications. The review focuses on uncertainty in metrology and accepted methods and standards for measurement system analysis. Specific application of the methods to the axle shim selection measurement processes is described. The chapter concludes with uncertainty methods specific to his study and the development of an uncertainty model.

Measurement System as Feedback

For maintaining a process under statistical control, Shewhart(1939) describes a continuing self-correcting process, in current terms, a feedback loop. Juran (1995) discusses the feedback loop as a simple model of manufacturing process control. This feedback loop provides information to adjust the process to maintain control relative to some criteria. The feedback model as described by Juran (1995) is applicable to any manufacturing process, even the craftsperson serves as process, sensor, umpire, and actuator in their work. An assessment system that is downstream and feeding back to the manufacturing process as illustrated in Figure 5. The sensor or monitor provides the necessary feedback to the umpire for decision making, that sensor is often some form of measurement system, the feedback serves as the process control method.

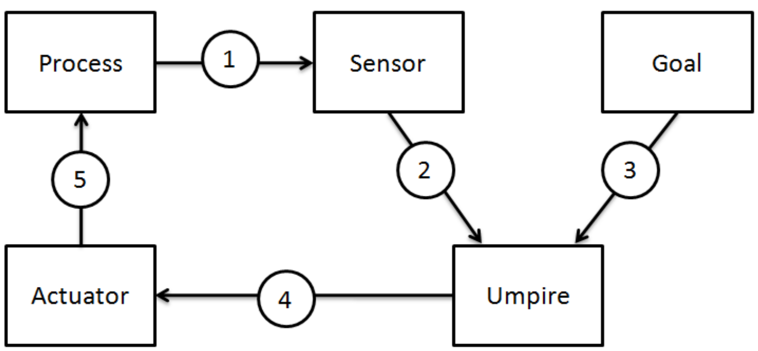

Figure 5. The feedback loop in generic terms (Juran, 1995, p. 617)

Achieving process control as discussed by Shewhart (1939) requires the removal of special or assignable cause variance, and monitoring of the process. This dictates the need for measurement of the manufacturing process artifacts i.e., a measurement system. Juran (1995) traces the history of measurements and the appropriateness of a measurement system from the ancients; he provides an example of quality control and measurement systems in use in China 3,000 years ago. The common understanding of measurement as described by Grubbs (1948) is a two-component concept, one the true or absolute value and the other the measurement error.

Grubbs (1948) was one of the first to discuss the application of statistical methods to discern the error of the measurement system, describing methods to estimate variance based on multiple measurements.

As has been stated earlier, the domestic automotive companies of Chrysler, General Motors, and Ford combined to publish a manual under the auspice of the Automotive Industry Action Group (AIAG) for assessing measurement systems in 1990. The AIAG *Measurement System Analysis (MSA)*(1990) applies statistical methods as part of the operational definition of a measurement system, and requires that the measurement variability must be small relative to the process and specification limits (p. 5). The International Standards Organization (ISO) more recently published Standard *22514*-7, *Statistical methods in-process management – Capability and performance – Capability of a measurement process*(ISO, 2012). In both of these recognized industry standards, assessment of the measurement system is relative to limits of a manufactured feature. Both standards discuss the suitability of a measurement process, AIAG MSA describes the first step of measurement system development is to establish the purpose and how it will be utilized (AIAG, 2002, p. 24). The ISO (2012) measurement system capability standard similarly states that the measurement process must be suited for a specific measurement task. The assessment metric, whether MSA(AIAG, 2002)GR&R results or the ISO (2012) Performance Ratio and Capability Indices, all are referenced to specification limits. Neither provides a specific method to assess a measurement system used for the shim selection process.

Statistics in Measurement Systems

Measurement accuracy and precision as discussed by Shewhart (1939) emphasized the collection of enough data to make an accurate decision. The purpose for this accuracy and precision for Shewhart(1939) was in support of maintaining a process in statistical control. He emphasizes repeated measures as a requirement to provide a basis for confidence in future measurements. Another Shewhart(1939) contribution to expressing measurement accuracy is the concept of quantitative and qualitative elements of a measurement process. Shewhart(1939) discussed measurement and accuracy in reference to statistical control and the expression of limits but does not address accuracy assessment of the measurement device. The concept of measurement as a combination of the absolute value of the characteristic being measured and the error of the measurement is the basis of Grubbs (1948) publication on precision and reproducibility of a measurement instrument. Grubbs (1948) defines the concept with a simple expression, $measurement = x_i + e_i$, where x_i is the true-value and e_i is the measurement error. The expression provided the starting point for Grubbs (1948) derivation of separate variances for the measured feature and the measurement error. Grubbs (1948) further derived methods for isolating the measured feature variance from the measurement error through multiple measurements; the method later adopted in Gauge Repeatability and Reproducibility (GR&R) studies.

Shewhart is credited by Harry Ku et al. (1969) with introducing statistical control on the quality of manufactured items, but Ku credits Pontius and Cameron (1967) with the first published example of applying statistics to the expression of the precision of a measured value. The concept of uncertainty in measurement, and the application of repeated observation sample error as an estimate of standard deviation to express this uncertainty, is included in Pontius and Cameron (1967) published research. The combined measurement uncertainty described by Pontius and Cameron (1967) included subjective bias estimate as systematic error, and estimated standard deviation as random error.

The expression of uncertainty that includes separate expression of systematic error, accuracy, or bias, from the random process error is an element of Eisenhart (1963) publication on instrument calibration. Among the methods Eisenhart (1963) included was an unbiased estimate of the within-occasion standard deviation from the average range ($\bar{R}$), that is applied in constructing Range Charts. The estimation of standard deviation from a series of samples based on range was developed by Patnaik (1950) as an efficient and simplistic practical statistical tool for small sample sizes. Patnaik (1950) derives a constant d_n used to estimate standard deviation based on an average range of groups sample measurements where, σ_w = unbiased estimate of standard deviation for a sample $\bar{R}$ = sample average range, d_n = unbiased estimator constant.

$unbiased\ estimate\ of\ \sigma_w = \bar{R}/d_n$

The early applications of assessing measurement error, or uncertainty, focused on standards and laboratory calibration, a result of increased accuracy requirements in the missile and satellite fields (Ku et al., 1969). The expression of imprecision and systematic error as a clarifying statement to a measurement result is the approach adopted by Harry H. Ku (1969). Ku outlined techniques for expressing imprecision and systematic error or bias, and included details on the expression uncertainty. Ku (1969) included the concept of error influences based on judgment as part of the uncertainty including expressions like “believed”, “estimated”, and “considered”.

The extension of statistical methods from laboratory to metrology for manufacturing environments is discussed by Ku (1967). He applied statistical concepts in clarifying measurement uncertainty to include computing confidence intervals and limits. The two underlying assumptions in analyzing a measurement process are identified by Ku (1967), that the measurement follows the normal distribution, and a reliance on the Central Limit Theorem. The

Central Limit Theorem assumption leads to a confidence interval when the variance σ^2 of the population is known; $\sigma_M = \sigma/\sqrt{N}$ for N measurements, where σ_M is typically referred to as the Standard Error of the Mean (Warner, 2013). Ku (1967) includes a discussion of sample mean $\bar{x}$ and sample variance s^2 as a method to estimate the standard error of the mean, expressed as $SE_M = s/\sqrt{N}$ to clarify it is based on the sample variance. The methods outlined by Ku (1967) include estimating confidence intervals when the population variance is unknown by calculating the variance of the measurements. This approach is included in Warner (2013) statistics text applying Students t distribution as an assessment of upper and lower confident limits on a measurement applying $t_{critical}$ probabilities where degrees of freedom (df) are based on sample size (*N*) as, $df = N - 1, Measurement\ confidence\ interval = M \pm (t_{critical} * SE_M)$

Expressing uncertainty in measurement that includes various contributors beyond the measurement variance is emphasized by Eisenhart (1968) to include detailed descriptions of standard error and degrees of freedom from multiple sources in reporting results. The method to propagate errors in metrology when multiple measurements are applied is a topic Ku (1967) includes in his literature. The general formula for error propagation with two measurements from Ku (1967) where s_i represents the standard error of the measurement data;

$$s_w^2 = \left[\frac{\partial f}{\partial x}\right]^2 s_x^2 + \left[\frac{\partial f}{\partial y}\right]^2 s_y^2 + 2\left[\frac{\partial f}{\partial x}\right]\left[\frac{\partial f}{\partial y}\right] r_{xy} s_x s$$

Provided the measurements are independent $r_{xy} = 0$ and the covariance does not contribute to the propagated variance. This approach provided a method to apply uncertainties in measurements to a reported result. Another established method Ku (1967) applied to early metrology, estimating standard deviation based on the range of a group of measurements, a method that was applied to statistics by Patnaik (1950) and later adopted by AIAG MSA for average range GR&R.

Uncertainty Analysis

The concepts of measurement error and measurement uncertainty included in publications on laboratory results, metrology, and measurement systems were the subject of numerous publications, including the National Bureau of Standards collection by Ku et al. (1969), but a broadly accepted approach to express measurement error and uncertainty did not exist. This issue was acute in the aerospace industry, as Abernethy, Benedict, and Dowdell (1985) identified that the absence of an uncertainty calculation standard made results comparison difficult. The publication by Abernethy et al. (1985) maintained the basic principle of expressing error as systematic or bias error, and precision or random error. Adapting the standard error formula, as did Pontius and Cameron (1967) for random error, Abernethy et al. (1985) identified that the bias error had no statistical equation and must be estimated. Conceding the judgmental nature of bias error estimate, Abernethy et al. (1985) combined the two errors as an estimate of the measurement uncertainty (U) using the expression $\bar{X} \pm U$, where $\bar{X}$ represents the average measurement and U is the associated uncertainty. Adopting Student's t statistic in the same manner as Ku (1967), the authors identified two confidence interval expressions for uncertainty. This became an accepted approach to quantify both statistical and expert judgment of measurement error. A unified approach for expressing uncertainty was finally introduce with the publication of the *Guide to the expression of uncertainty in measurement* (GUM) in 1993 (JCGM, 2008a).

The common application of uncertainty in reference to the quality of measurement includes two factors as identified by Kessel (2002), the equality of the measurand true value to the measurement, and the accuracy of that measurement. Kessel (2002) refers to the GUM as outlining the unified approach to stating the uncertainty in measurements. The GUM (JCGM, 2008a) is a publication by a working group of experts selected by the Bureau International des Poids et Measures (BIPM), the International Electrotechnical Commission (IEC), the International Organization for Standardization (ISO), and the International Organization of Legal Metrology (OIML). Virtually every source, including MSA(2010), ISO (2005), Coleman and Steele (2009), Hughes and Hase (2010), and Taylor and Kuyatt (1994)NIST Guidelines, refer to the GUM as the primary reference in uncertainty analysis. The concept of quantified uncertainty by GUM (2008a) is an expression of the combined error, established through error analysis, of the uncertainty or doubt about the stated result. The GUM (2008a) provides methods for evaluating and expressing how well the result of the measurement represents the true value of the measurand.

The methods for developing uncertainty expressions follow what Shewhart (1939) described, quantitative and qualitative elements. The contribution of the GUM (2008a) is in providing methods for evaluating and expressing measurement uncertainty that are a consensus of recognized international organizations. Prior to the publication of the GUM (2008a) the concept of a measurement as an expression of the true value and measurement error that includes statistical and judgment elements was well established. Taylor and Kuyatt (1994)NIST Technical Note discuss that these two recognized elements of measurement error are classified and combined in two categories of uncertainty, consistent with GUM, as Type A, that evaluated by statistical methods, and Type B, that evaluated by other means. The expression of Type A and Type B uncertainties are part of the measurement lexicon, as Coleman and Steele (2009, p. 264) state, to remove the ambiguity associated with the previous expressions of systematic and random error.

The sources of Type B uncertainties are both random and systematic effects on measurements. ISO (2005) lists examples of Type B evaluations including, calibration of reference standards, environmental effects that cannot be sampled, misalignment in the measurement instrument, and instrument resolution. The technique identified by ISO (2005) is to estimate the worst case based on experience, scientific judgment, and scant data. The GUM (JCGM, 2008a) describes Type B estimates of uncertainty as scientific judgment based on "the pool" of available information, including "experience with or general knowledge of the behavior and properties of relevant materials and instruments" (p. 11). The GUM (JCGM, 2008a) defines that the purpose of Type A and B classifications is to indicate the method used, both are based on probability distributions and that Type B methods can be as reliable as Type A methods.

Guidelines for applying Type A and Type B uncertainty methods are provided in three publications that reference the GUM. The ISO (2005) Technical Specification 21749:2005, *Measurement uncertainty for metrological applications – Repeated measurements and nested experiments*, applies the principles to ongoing manufacturing processes. The ISO (2012)22514-7, *Statistical methods in-process management Capability and performance, Part 7: Capability of measurement processes*, applies the principles to measurement system capability analysis. Finally, Dietrich and Schulze (2011)*Measurement Process Qualification, Gauge Acceptance and Measurement Uncertainty According to Current Standards*, applies the principles of the Guide and ISO standards to manufacturing applications.

Measurement uncertainty analysis applies the GUM (2008a) expressions for standard uncertainty, $u(x_i)$, and standard variance $u^2(x_i)$, for both Type A and Type B uncertainties. To determine a combined uncertainty $u_c(y)$ and variance $u_c^2(y)$of a measurement process the individual measurement uncertainties are summed using GUM Equation (10).

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i)$$

Where f is a functional expression of the measurement described by GUM Equation (1).

$$Y = f(X_1, X_2, \dots, X_N)$$

The ISO 22514-7 (2012) provides a method to apply the GUM equation to combine individual component uncertainty variance u_i^2 to determine a combined measurement variance u_{Meas}^2 contributors to uncertainty;

$$u_{Meas}^2 = \sum_{i=1}^{N} u_i^2$$

Monte Carlo Uncertainty Methods

A statistical method to study complex physical phenomena using random number generation was introduced to literature by Metropolis and Ulam (1949) which they referred to as the Monte Carlo Method (MCM). The concept of applying statistical techniques to problem solving was known prior to this publication but was not commonly used due to the tedious calculations involved. The computational power of computers revived the approach to solve complex problems; scientists at the Los Alamos Laboratory were first to apply the Monte Carlo name(Metropolis, 1987). Monte Carlo Simulation (MCS) is a tool commonly applied in measurement uncertainty analysis to propagate multiple error sources to determine the combined standard uncertainty. Coleman and Steele (2009)outline the simulation process used to incorporate error from multiple sources in predicting the output variable distribution, shown in Figure 6. It is initiated by selecting a probability density function (PDF) to produce the true value, then adding the error from the various sources based on their individual PDF's. The MCS inputs include selecting the appropriate PDF and the number of trials for the simulation.

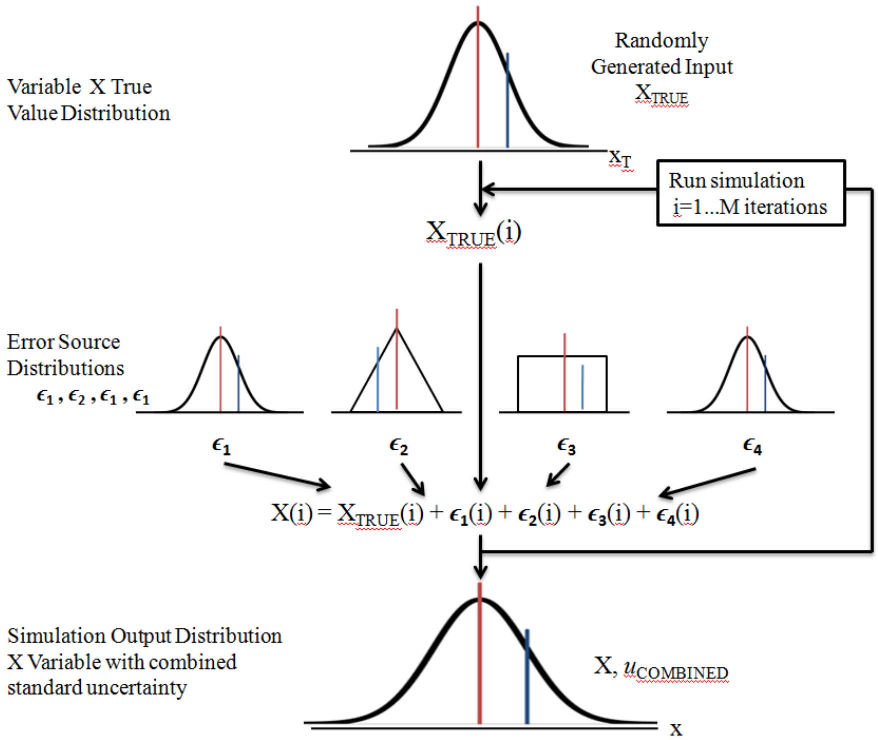

Figure 6. Monte Carlo Method for combined standard uncertainty (Coleman & Steele, 2009)

Previous studies have applied MCM in measurement system analysis, Yeh and Sun (2013) applied MCM to estimate the variability of GR&R. They provided the basic steps in an MCM study as selecting the appropriate PDF to generate the random input, propagate the inputs through the model to generate an output, repeat the steps for the number of iterations, and analyze the results. The number of simulation iterations is based on the purpose of the analysis that is commonly to determine the output reported coverage factor. Couto, Damasceno, and de Oliveira (2013) discuss methods for determining MCM iterations including following the GUM guidelines for number of trials $M > 10^4/(1-p)$ where p is the probability coverage for the output, and convergence methods that monitor the stability of the output. Coleman and Steele (2009) approach for determining MCM iterations for standard uncertainty is to monitor the standard deviation of the output and stopping iterations once it converges to a limit based on the purpose of the analysis.

The application of the MCM is common in many research fields because of the ease of use and efficiency (Kroese, Brereton, Taimre, & Botev, 2014). There are numerous software packages available for MCS, including Microsoft Excel®, which is commonly used due to the ease of programming and software availability (Farrance & Frenkel, 2014). McCullough and Heiser (2008) have criticized the use of Excel for statistical analysis as inadequate for statistical and scientific purposes, particularly versions prior to 2007. Farrance and Frenkel (2014)argue that Microsoft Excel® for MCS is an acceptable approach for analysis similar to uncertainty analysis, where heavy use of the random number generator feature is not required.

The axle shim-selection process includes multiple uncertainties with varying PDF's, this fits the Coleman and Steele (2009) feasibility criteria for the application of MCM uncertainty analysis. A shim-selection MCS is an effective method to propagate multiple uncertainties for each input and output. An advantage of MCS is the ability to generate a large amount of simulated production data that matches the actual production process. The simulated data can then be directly compared to production data using data visualization and accepted statistical techniques. This includes the ability to compare input and output variables, and in-process observations to validate the analysis. Another benefit is the flexibility of a MCS model. With an MCS model, changes in variables and in the process can be readily made and compared. A Microsoft Excel® shim-selection process MCS is a convenient and accurate method to correlate input and output variables.

Industry Approaches to Measurement Uncertainty

Early examples of published uncertainty analysis methods focused on the laboratory and testing facilities, for example the Abernethy and Thompson Jr. (1973)*Handbook, Uncertainty in Gas Turbine Measurements*. The emphasis was on reporting and communicating specific confidence ranges in measurement testing results and scientific data. An early treatment of uncertainty in an industrial application is by Ku (1967) in the American Society of Tool and Manufacturing Engineers *Handbook of Industrial Metrology*. Ku (1967) described the measurement process and assessing the variability using statistical methods. In the automotive industry, measurement error is discussed in the first edition of the AIAG MSA Manual, but the topic of uncertainty is not discussed until later editions. The third edition of MSA(AIAG, 2002) discusses the topic of uncertainty in two contexts. The first is a general definition as the range of measured values in which the true value is contained (p. 8). The second context is the more formal uncertainty analysis. The reference manual identifies the concept but specifically delineates that MSA focuses on understanding the measurement process, and defers to the Guide to the Expression of Uncertainty in Measurement (GUM) for the topic of uncertainty, claiming it a high-level reference document. GUM defines uncertainty in measurement as a reflection of the lack of exact knowledge of the measurand value (JCGM, 2008a, p. 5).

An alternate standard that provides insight into the application of uncertainty in industrial applications is ISO (2012) 22514-7 *Statistical methods in-process management Capability and performance Part 7: Capability of Measurement processes*. This standard provides a statistical approach to assessing uncertainty and applying that to assessing capability of measurement processes. The contrast between MSA and ISO approaches are made by Dietrich (2014), noting that ISO guidelines describes assessing a measurement system, including uncertainty from test part variation, separate from influences not directly the result of the measurement system. Table 1 summarizes the ISO (2012) defined components of uncertainty, identifying seven categories of uncertainty. The application of the uncertainty expression by ISO (2012) provides greater granularity than MSA GR&R, defining two performance ratios and two performance indices for measurement system assessment.

Table 1

ISO 22514-7:2012 Standard uncertainty component descriptions

Component	Symbol	Description
Maximum Permissible Error (MPE)	u_{MPE}	A standard and known uncertainty expressed as the probability of a rectangular distribution: $$u_{MPE} = \frac{M_{PE}}{\sqrt{3}}$$
Measurement System Resolution	u_{RE}	One half measurement resolution R_E expressed as the probability of a rectangular distribution: $$u_{RE} = \frac{1}{\sqrt{3}} * \frac{R_E}{2}$$
Calibration	u_{CAL}	Standard deviation of uncertainty due to calibration, including the coverage factor k: $$u_{CAL} = \frac{U_{CAL}}{k_{CAL}}$$
Linearity	u_{LIN}	Uncertainty resulting from linearity. Determined from a calibration certificate, experimentally by ANOVA methods, or from uniform distribution range where *a* is half width of the uniform distribution or known MPE value: $$u_{LIN} = \frac{a}{\sqrt{3}}$$
Bias	u_{BI}	Difference from a measured standard: $$u_{BI} = \frac{\lvert \overline{x_g} - \overline{x_m} \rvert}{\sqrt{3}}$$
Reference Standards	u_{EVR}	Minimum of 30 repeated measures on a reference standard or standards. The uncertainty is the unbiased estimate of the standard error: $$u_{EVR} = \sqrt{\frac{1}{K-1}\sum_{i=1}^{K}\left(x_i - x_g\right)^2}$$
Other uncertainty components	$u_{MS-REST}$	Other uncertainty components not characterized above that influence the measuring system.

The application of uncertainty analysis by ISO (2012) includes two acceptance criteria, one a capability index C_{MS}, and a capability ratio Q_{MS}. Comparing the ISO 22514-7 standard to AIAG MSA Dietrich (2014) contends that the ISO capability assessment process has an advantage. The separation of uncertainty into component elements of the measurement process that affects the uncertainty permits one to assess the source of uncertainty independently. Dietrich (2014) contends that this is enough of an advantage to select the ISO 22514-7 approach over AIAG MSA, but concedes that the MSA global recognition and ease of application provide arguments for that approach. Either AIAG or ISO approaches apply the same evaluation techniques, an assessment of the measurement system to the feature specification limits. MSA evaluates the ratio of the repeatability and reproducibility (GR&R) relative to the total variation (TV) of the upstream process.

$$\%GRR = 100 * \frac{\sqrt{(EV)^2 + (AV)^2}}{TV}$$

The assessment method of ISO 22514-7 includes the upstream process upper and lower specification limits in the assessment metrics. The performance ratio Q_{MS} evaluates the uncertainty as the percentage of the specification range upper limit (U) and lower limit (L).

$$Q_{MS} = \frac{2 * U_{MS}}{U - L} * 100$$

Similarly, the ISO Capability index C_{MS} is related to the specification range.

$$C_{MS} = \frac{0.3 * (U - L)}{6 * \hat{u}_{MS}} * 100$$

Measurement Requirements for Precision Assembly

The assembly of manufactured components with interchangeable parts started, as Shewhart(1939) describes, as an exact science, assuming individual assembly components are manufactured to exact dimensions. The statistical process control concept

Shewhart(1939)introduced, provided a method to analyze and control the manufacturing process but the responsibility of assigning the limits to manufactured components fell on the design engineer. The design engineer had to assign tolerances that resulted in acceptable assemblies, and to perform analysis that determined tolerance limits. The concept of an assembly tolerance stack and component tolerance allocation Scholz (1995) traces back to 1925 with numerous publications in the 1950's on the topic. The two tolerance allocation methods Scholz (1995) summarizes are arithmetic and statistical tolerancing. The arithmetic tolerance assumption is that the component feature can have any value within the tolerance range and that all possible values are within that range. The statistical tolerancing approach assumes that the component features vary following a distribution, most commonly a normal distribution. The intuitive issue for assembly of interchangeable parts as Scholz (1995) described is the inability to assemble parts considering each relevant feature includes variation.

The goal of allocating the tolerance of components in an assembly as summarized by Scholz (1995) is to maintain the allowable assembly error within the detail component manufacturing errors. Ultimately, the purpose of tolerance allocation is one of cost associated with part and assembly manufacture, to the extent one can relax tolerance the cost to manufacture the assembly is reduced (Scholz, 1995). Researchers have developed alternate methods for allocating and analyzing tolerances. Greenwood (1987) proposed alternate approaches to statistical tolerancing to predict the outcome of an assembly process modeling the component process bias along with the random variation. Chase and Parkinson (1991) summarized three methods for tolerance allocation, worst case (arithmetic), statistical, and simulation. Heling, Aschenbrenner, Walter, and Wartzack (2016) propose an integrated model that includes manufacturing cost in the allocation of tolerances. The research on tolerance allocation has a common theme described by Chase (2004), as manufacturers pursue higher quality products, controlling and monitoring manufacturing variation consumes much effort. The purpose of manufacturing tolerance studies is the allocation of tolerance at the component level. Chase (2004) describes the process as predicting accumulated variation in the assembly and driving the process to manufacture the components within statistical control to meet the assembly requirements.

Reducing process variation to simultaneously improve quality and reduce cost is a phenomena recognized by Dr. Genichi Taguchi (Sullivan, 1987), who developed a method to analyze the relationship. Taguchi's "Loss Function" $L(x)$ assesses the monetary impact of process deviation. It is used to calculate the loss a when variable x is not at the target value T by magnitude $b, L = (a/b^2) * (x - T^2)$(Drake, 1999). The concept applied to a normal distribution is shown in Figure 7. The method can be applied to a manufacturing process to make trade-offs based on the monetary impact of deviation.

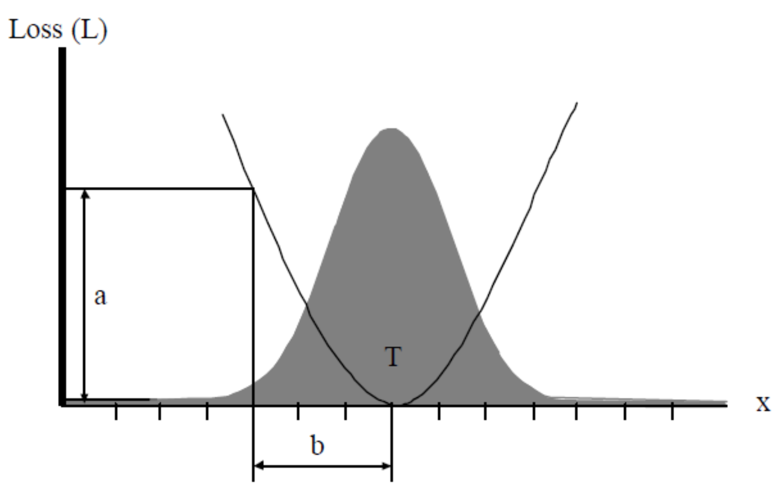

Figure 7. Taguchi loss function relative to a normal distribution (Drake, 1999)

The method to communicate manufacturing requirements includes dimensions and tolerances as part of engineering drawings. One of the earliest drawing standards was issued in

1927, British Standard No. 308, the first American standard for drafting was issued in 1935 by the American Society of Mechanical Engineers as Dimensional and Tolerancing Standard Y14.5 (Srinivasan, 2008). The standards have been revised since inception and now adopt the expression Geometric Dimensioning and Tolerancing (GD&T) to communicate requirements. The application of GD&T requires the designer to transfer the design requirements into unambiguous and measureable specifications (Drake, 1999). The application of GD&T in components that make up complex assemblies is to improve the assembly process. Although three-dimensional models of components and assemblies commonly communicate nominal geometry, the tolerance requirements are typically communicated in two-dimensional drawings, what Srinivasan (2008) refers to as dual product documentation. Complex assemblies and feature requirements such as torque and gear backlash typically remain as minimum and maximum drawing limits.

Complex assemblies where component manufacture under statistical control limits to satisfy precision downstream criteria is not economical require an alternate approach. Automotive axles are an example where manufacturing the individual components within tolerance limits to achieve assembly requirements is impractical. The axle assembly of this study limits backlash to ±0.050 mm, equivalent to a ring gear positional tolerance of ±0.075 mm. An acceptable manufacturing process would require the allocation of this tolerance across multiple machining processes, gear manufacture, welding, and the two tapered roller bearings. Just the two automotive quality tapered roller bearings require ±0.200 mm, more than the total permissible tolerance. To meet the assembly requirements in many axle designs select-fit spacers or shims are included in the assembly to accommodate the manufacturing variation of the components. Selecting this shim requires inspection and recording measurements of individual components. Shims are classified, typically in 25-micron increments, and the process selects the shim class that most closely meets the requirement for the specific axle being assembled.

Automotive Axle Assembly

As stated earlier, in automotive axle assembly, hypoid gears are common and accurate positioning of hypoid gears is critical for providing durable and low noise producing assemblies (Spear & Baxter, 1966). Accuracy in setting pinion and ring gear position is the most challenging process in axle assembly. The arrangement of hypoid gears is such that the pinion axis position relative to the ring gear axis-centerline varies in two directions. A common method of describing this relationship is the P-E-G system, illustrated in Figure 8. The G dimension defines the gear axial position along the gear centerline relative to pinion centerline. The E dimension is the off axis distance or offset of the pinion center from the gear centerline perpendicular, often referred to as the hypoid offset. The P dimension is the position of the pinion along the pinion axis relative to the ring gear centerline. In automotive axle assembly, the design commonly includes the flexibility to vary the pinion position P and gear position G through selectable shims or other methods. In axle assembly, the gear position selectable shims are referred to as differential or case shims signifying that they position the differential case(Spear & Baxter, 1966).

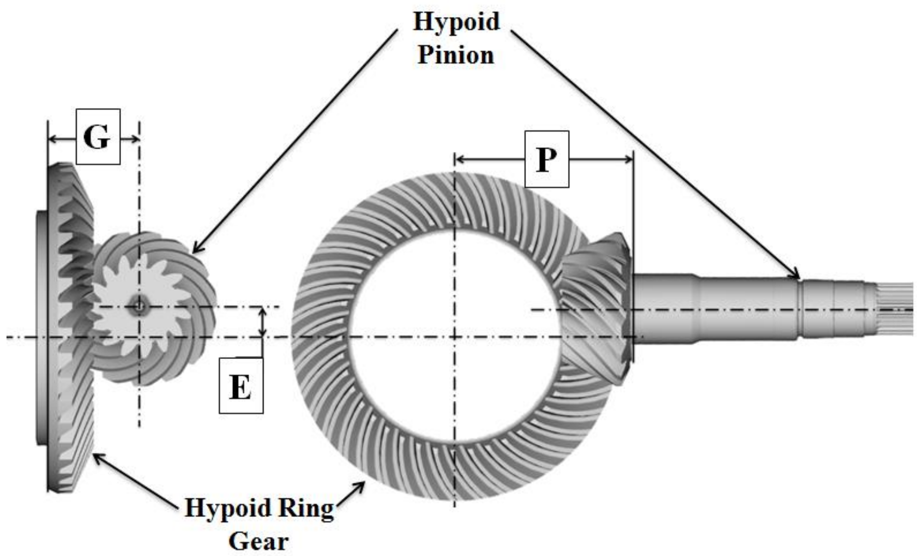

Figure 8. Axle hypoid gear positioning P-E-G system

A common manufacturing method for automotive hypoid gears includes lapping of mating pinion and ring gears after heat treatment. Once the gears are lapped, they are serialized and maintained as a match set. After lapping the matched set gears are roll tested to validate the gears are correctly manufactured by validation of the contact pattern. The technique used to roll test the matched gears is termed Single Flank Testing (SFT) signifying the gear teeth are evaluated separately on the convex flank, or tooth side, and the concave flank (Stadtfeld, 2011b). The SFT validates gear performance under load including information on transmission error. The SFT process positions the Pinion at the proper mounting distance “P” and the ring gear in contact with the pinion while rotating. The gear is retracted a known distance that positions the ring gear axially away from the pinion, creating backlash. This position is measured and the deviation from the theoretical “G” is recorded. That deviation from the ideal “G” is defined asδJ, a value recoded in millimeters and tracked for each gearset as shown in Appendix A, Figure 26.

In addition to gear positioning, the requirement to maintain axial load on the bearings contributes to the gear positioning system complexity. The system elements consisting of the bearings, carrier or housing, and differential case are a series of mechanical components that react to load in the same manner as a mechanical spring. Using select fit shims as a method to position the gears was originally introduced by Boden (1936). The current approach is to assemble the shims between the bearing outer-race, commonly referred to as the cup, and a machined surface on the carrier. The equations used to calculate the required shim thickness introduced in Chapter 1 are repeated below.

$$Pinion\ Side\ Shim\ thickness = CAR1 - CAR2 + (G + \delta J) - (OAH - BF) + OFFSET_{PS} \tag{1}$$

$$Gear\ Side\ Shim\ thickness = CAR1 + COV - OAH - PS_{Meas} + OFFSET_{GS} \tag{2}$$

A diagram depicting an axle measurement system identifying in-process and audit gauges is shown in Figure 9. The specific process for selecting the shims varies, but typically includes a series of in-process measurements of the components. At the end of the assembly process there are two measurements used to verify the shim selection process, these measurements serve as an audit of the process. The first audit is gear backlash used to assess the position of the gears. The second is rolling torque measurement used to assess bearing axial load. The result is a mass produced product that each individual assembly is comprised of a specific group of components individually measured in a measurement system integral with the assembly process.

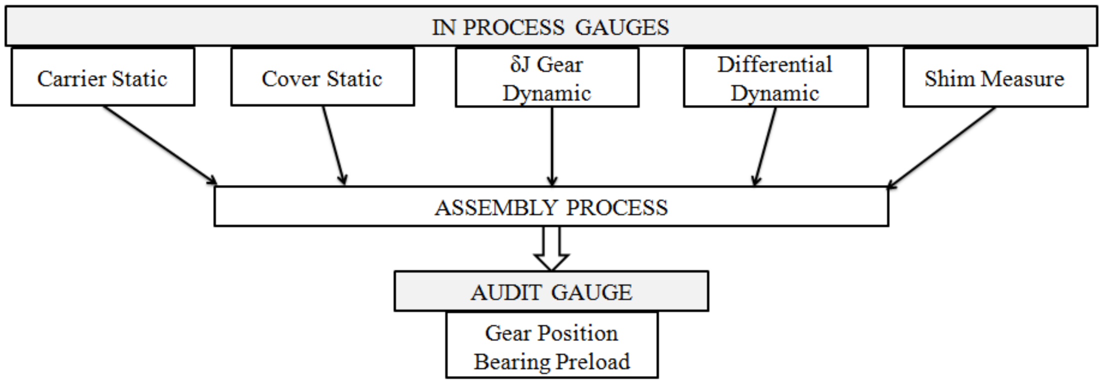

Figure 9. Automotive axle-assembly measurement system process diagram

The accepted concept of measurement, as described by the Bureau International des Poids et Measures (BIPM, 2008b), is that the measurement estimates the true quantity value that is constant and unknowable in practice. However, in case of automotive axle assembly the true value is not constant. To obtain an estimate of the true value Hughes and Hase (2010) identify repeated measures as the best method to average out errors, a technique not practical in high volume production environments. Test article variation, or within part variation, is acknowledged by MSA(2010) as a source of variance, and that ANOVA, Range, and Average Range methods all ignore this factor. This artifact variation differs from Part Variation (PV) that MSA defines as expected part and time variation for a stable process (p. 216).

In the axle assembly process, parts that are a source of within part variation that contribute to measurement error are the bearings and the hypoid gears. The bearing geometry, precession rates, and effects of lubrication all affect the position of the bearing races (height) and the measured torque (Timken, 2011). Bălan, Stamate, Houpert, and Olaru (2014) identify that this variation is complex and highly dependent on lubrication such that compensation through algorithms in an assembly process is not practical. Hypoid gear design in automotive axles seeks to minimize the frequency that the same two teeth mesh on the mating gears, defined as the hunting tooth frequency. As noted by Stadtfeld (2014) hunting tooth design requires one full revolution of the pinion for each ring gear tooth to characterize backlash fully. Measuring all tooth combinations in a production environment is impractical. Currently there is no concise method to include known part variation in measurement system analysis.

The assessment of within part variation in a measurement system is part of the ISO 22514-7 (2012) standard, it is described as non-homogeneity of the part and is symbolized as u_{OBJ}. ISO standard guidance on assessing the non-homogeneity centers on feature variation and assumes a rectangular distribution for predicting the uncertainty based on product drawing or experimentation. The AIAG MSA analysis does not include a method to distinguish part variation from the random error of the measurement system to that of the manufacturing process. The ISO 22514-7 identifies it as an uncertainty in the measurement system capability but bases that uncertainty prediction on generalized assumptions.

Summary

A review of the literature provides the background for the application of measurements and the uncertainty of measurements. In more complex measurement systems, like axle shim selection, the process is affected by measurement errors, uncertainty associated with part variation, and uncertainties not directly measured. An approach for expressing and analyzing measurement errors is uncertainty analysis, a concept introduced to metrology by Ku (1967) and codified in the Guide to Measurement Uncertainty published by JCGM (2008a). Uncertainty analysis provides a method to include various sources of error and variation to predict the outcome in a measurement process. ISO (2012) has published guidelines for uncertainty analysis specific to manufacturing in the Capability of measurement processes Standard 22514-7. The ISO Standard does not include methods for analyzing complex systems, like axle shim selection, but when combined with other literature and Monte Carlo Methods an approach for modeling and analyzing complex measurement systems is possible (Coleman & Steele, 2009).

The research methodology presented in the following chapter outlines a quantitative study that Creswell (2013) describes as “a means for testing objective theories by examining the relationship among variables.” Following Creswell’s (2013) definition of the theory of quantitative research as the use of interrelated variables, in this study measurements and their uncertainties, formed into propositions that specify the relationship. The proposition for this study is an uncertainty model explains the correlation of input measures and resulting audit measures. To validate the theory experimental results from a production manufacturing system are compared to results from a Monte Carlo simulation(Coleman & Steele, 2009).

The task for manufacturing systems technology management as summarized by ISU (2016) is the application of technology to add value to a manufacturing process, and profitably produce products. Standards and publications for measurement system selection and analysis emphasize appropriateness, MSA(AIAG, 2002, p. 24), ISO 22514-7 (2012), Dietrich and Schulze (2011) all concur that the measurement process must be appropriate for the specific task. Uncertainty analysis provides an appropriate approach that includes a mix of uncertainties evaluated by statistical methods and those evaluated by other means. A predictive technique is the basis for meaningful measurement system acceptance criteria when upper and lower limits do not apply. Further, a method to correlate input measurements to process outputs can be used to predict a priori the performance of an axle assembly process thereby providing management with a tool to support decisions on the measurement and assembly system.

CHAPTER 3

METHOD OF INVESTIGATION

Overview

This study evaluated a method to correlate input measurements to audit measurements of an axle shim-selection system. The assembly and shim-selection measurement system shown previously in Figure 3 and Figure 9 identify the elements of the process selected for this research. The axle shim-selection process includes factors that are not part of the assembly and measurement system. To include the effects of all of the factors that influence the process, the technique of measurement standard uncertainty was applied. The standard uncertainty of each independent and dependent variable was assessed and then propagated through the process. To propagate the standard uncertainties a Monte Carlo Simulation (MCS) was developed in Microsoft Excel® 2007 (12.0.6214.1000).

The study methodology included three stages. The first stage defines the combined uncertainty for each measurement variable and assembles the uncertainties into a model of the system. The second stage combines the uncertainties with simulated true values in a Monte Carlo simulation to generate simulated data of the selected shim selection system. The simulation data is compared to production data to evaluate the ability of the model as a correlation tool. The third stage applies the model to assess the effects of measurement capability for a specific independent variable on the performance of the measurement system.

To answer Research Question 1, how can measurement uncertainty methods be applied to model the axle shim-selection measurement process, an uncertainty propagation model was developed. Each measurement variable was individually analyzed for uncertainty following GUM guidelines and techniques that are summarized for manufacturing measurement systems by ISO (2012), Dietrich (2014), and Dietrich and Schulze (2011). The component uncertainties include both Type A and Type B standard uncertainties. Type A uncertainties are based on observation data and analysis using statistical methods that estimate the standard uncertainty. Type B methods are based on non-statistical methods including published literature, engineering analysis, or known relationships. The component uncertainties for each measurement determine a combined standard uncertainty for that measurement. The combined standard uncertainties were propagated through the shim call equations, Equation 1 and Equation 2 in Chapter 1, following the approach initially published by Ku (1967) and included in the GUM. The propagated uncertainty results were documented in tabular form for each variable.

To answer Research Question 2, Can a measurement system uncertainty model be used to predict the backlash and torque-to-rotate capability of a shim-selection measurement system, the study evaluated the model effectiveness by comparing predicted results with sample data from a production system using a Monte Carlo simulation. The uncertainties developed as part of Research Question 1 were included in the simulation to produce a set of virtual data that was compared to sample production data. To obtain the sample production data, the factory Supervisory Control and Data Acquisition (SCADA) was used. SCADA data for a continuous production of two-thousand sequential parts was collected on an axle assembly line running three unique models. The data was qualified, separating cases by axle type and excluding any rerun axles or axles with incomplete data sets. Statistical methods were then used to identify and eliminate outliers. The production data is statistically compared to the virtual model data to identify the ability of the model to predict the results, distribution, and process capability.

To answer Research Question 3, can a measurement system uncertainty model be used to determine the acceptance limits for an individual in-process shim-selection measurement apparatus, the study assessed a selected static measurement that demonstrates stable process capability. The analysis of that independent variable compared results with an in-process measurement GR&R of 10% with an in-process measurement GR&R of 50% using AIAG MSA methods based on the drawing tolerance. The Monte Carlo simulation produced two separate data sets. A comparison of the two data sets, analyzing the effects of measurement system capability on the shim selection process, provides an example application of the uncertainty model.

Study Variables

The measurement variables included in the axle shim-selection process are described in Table 2. The variables are classified as predictor input variables and audit variables; all variables are continuous. The input variable measurements are included in Equations 1 and 2 shown in Chapter 1 to determine the appropriate shim size for the assembly. The audit variable measurements are the outcome of the shim selection process. To support the uncertainty analysis the independent variable measurements were categorized as static dimensional measurements, dynamic bearing measurements, and gear position measurements.

Table 2

Shim selection process measurement variable descriptions

Measurement Description	Symbol	Measurement Apparatus
Predictor Input Measures (Independent Variables)		
Split Line to Pinion Side Shim Seat	CAR1	Carrier Gauge, Precision Static single purpose dedicated gauge using digital measurement probes
Split Line to Centerline of Pinion Head Bearing Bore	CAR2	
Split Line to Gear Side Shim Seat	COV	Cover Gauge, Precision Static single purpose dedicated gauge using digital measurement probes
Ring Gear measured difference from theoretical mounting distance "G"	δJ	"Single Flank" Transmission Error Gear Tester
Differential Assembly overall height Bearing cup to cup	OAH	Differential Gauge, Dedicated gauge that measures distance with probes while rotating the bearings under axial bearing load
Differential Assembly distance from Gear Side Bearing Cup to Gear mounting reference	BF	
Thickness of the selected shim	PS_{Meas} GS_{Meas}	Shim Verifier, Dedicated measurement device that measures the thickness of the selected shim
Audit Measures (Dependent Variables)		
Assembly Backlash measured dynamically	LASH	Dedicated dynamic audit gauge with precision encoder and torque transducer
Assembly Total Torque to Rotate measured at the Pinion	TTR	

Research Design - Question 1

Research Question One methodology applies measurement uncertainty to the axle shim-selection process to model independent input measures and predict dependent audit measures. The result is a prediction of the observed audit measurements standard deviation. The model development followed four steps as shown in Figure 10.

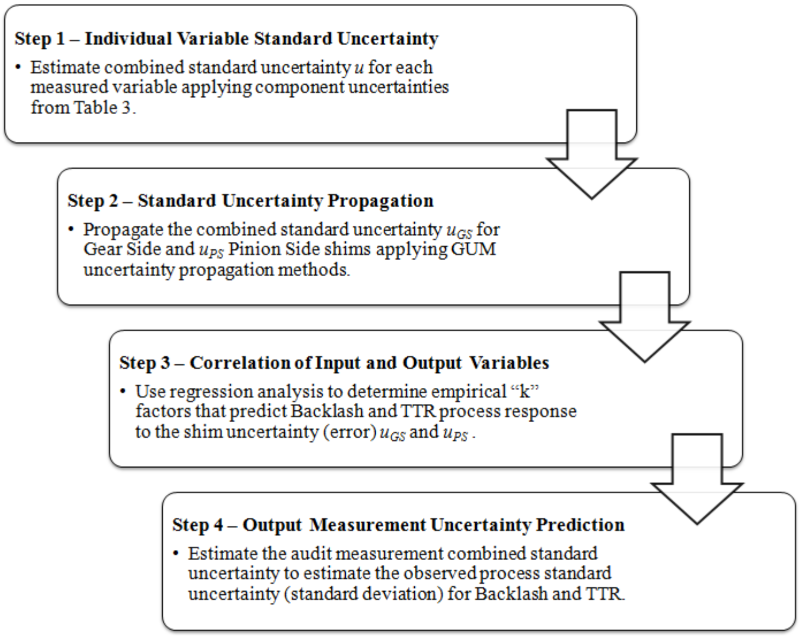

Figure 10. Design process steps for Research Question 1

Variable Standard Uncertainty Development

The first step combined individual standard uncertainty components, u_i to develop a combined standard uncertainty u_{Meas}, for each measured variable listed in Table 2. The components of standard uncertainty are part of the ISO (2012)*Statistical methods in process management – Capability and performance – Part 7: Capability of a measurement process*. The standard identifies fourteen uncertainty components for measurement processes and measurement systems. By combining the "other" category of measurement system $u_{MS-REST}$ with process u_{REST}, and excluding uncertainties associated with traceable standards u_{CAL}, and temperature variation u_T, eleven components were considered for each measurement variable. Table 3 summarizes the eleven components. The combined standard uncertainty for each measured variable was determined using Equation 3 below.

$$u_{Meas}^2 = \sum_{i=1}^{N} u_i^2 \tag{3}$$

The result of Step 1 are standard uncertainties for each measured variable; u_{CAR1}, u_{CAR2}, u_{COV}, $u_{\delta J}$, u_{OAH}, u_{BF}, u_{PSMeas}, u_{GSMeas}, u_{Lash}, u_{TTR} documented in tabular form in Chapter 4.

Table 3

Standard uncertainty components considered in this study (ISO, 2012)

Symbol	u Component	Comment
u_{LIN}	Linearity	Uncertainty arising from non-linearity of measurement
u_{BI}	Bias	Uncertainty resulting from bias relative to a standard
u_{EVR}	Repeatability on standard	Repeatability on a standard
u_{RE}	Resolution	Uncertainty based on the measurement apparatus resolution
u_{EVO}	Repeatability on workpiece	Repeatability on the workpiece based on repeatability studies
u_{AV}	Appraiser reproducibility	Uncertainty associated with operator influence based on repeatability studies
u_{GV}	System reproducibility	Reproducibility of the measurement system based on R&R studies
u_{STAB}	Reproducibility over time	Uncertainties associated with time based effects
u_{IAi}	Interactions	Uncertainties associated with appraiser related interactions from ANOVA studies
u_{OBJ}	Measurand non-homogeneity	Uncertainties arising from measurand non-homogeneity
u_{REST}	Other uncertainty contributors	Other uncertainty components that are not included above

Standard Uncertainty Propagation

The second step modeled the uncertainty associated with the Pinion Side Shim u_{PS}, and the Gear Side Shim u_{GS}. This step propagates the individual measurement standard uncertainties

u_i determined in Step 1 through the shim selection equation using the general law of error propagation (JCGM, 2008a). The Chapter 1 shim selection Equations 1 and 2, repeated below, include the measurement independent variables and an offset constant:

$$Pinion\ Side\ Shim\ Thickness = CAR1 - CAR2 + (G + \delta J) - (OAH - BF) + OFFSET_{PS} \quad (1)$$

$$Gear\ Side\ Shim\ Thickness = CAR1 + COV1 - OAH - PS_{Meas} + OFFSET_{GS} \quad (2)$$

The propagation of uncertainty applies what GUM (JCGM, 2008a) describes as "the law of propagation of uncertainty" (3.3.6). Specific consideration for this study is the propagation of uncertainty in the Gear Side Shim calculation for backlash. The carrier measurement $CAR1$ and the differential overall height OAH are included in the Pinion Side and Gear Side shim thickness calculation. The GUM method to treat this in uncertainty propagation considers the covariance of the variables. The uncertainty propagation to determine combined uncertainty of correlated variables includes interactions provided in Section 5 of the GUM, paragraph 5.1.2, equation (13) for uncertainty analysis.

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \frac{\delta f}{\delta x_i}\frac{\delta f}{\delta x_{ij}} u(x_i, x_j)$$

The PS_{Meas} applied to the Gear Side shim calculation includes measurement independent variables that are common with the Pinion Side shim-call equation. The GUM discusses uncertainties in terms of error, in this instance, the error would be duplicated and hence overstated. The two measurements are therefore not correlated, and should be treated as constants in the Gear Side shim calculation. The uncertainty propagation to determine combined uncertainty of uncorrelated uncertainties is provided in Section 5 of the GUM, paragraph 5.1.2, equation (10) for uncertainty analysis.

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i)$$

Applying the GUM method to analyze uncorrelated uncertainties to the Pinion Side shim call equation includes an assumption that there is no covariance of the individual measurements. For the Pinion Side Shim this assumption is valid as all measurements are independent. The GUM identifies the factors for each measurement variable as sensitivity coefficients. In the case of the Pinion Side Shim, all of the variables have unity sensitivity coefficients. There is an additional uncertainty included to account for the incremental shim steps u_{Step}^2. The resultant uncertainty estimate for the Pinion Side shim u_{PS} is.

$$u_{PS} = \sqrt{u_{CAR1}^2 + u_{CAR2}^2 + u_{OAH}^2 + u_{BF}^2 + u_{\delta J}^2 + u_{Step}^2} \tag{4}$$

The Gear Side shim call includes measurements common to the Pinion Side but no covariance. Two variables, CAR1 and OAH, are included in the Gear Side shim thickness calculation. These two variables are treated as constants in the Pinion Side uncertainty u_{GS} as it relates to backlash.

$$u_{GS} = \sqrt{u_{COV1}^2 + u_{PSMeas}^2 + u_{Step}^2} \tag{5}$$

Correlation of Input and Audit Variables

The third step determined the effects of shim selection uncertainty of Equations 4 and 5 on the audit process dependent variables using regression techniques. Shimmed axles respond to shim dimensional changes with an interaction of bearing preload force and backlash due to relative stiffness of the assembled components. As part of this study, the relationship was determined empirically by post processing data from re-shimmed axles. The response of backlash and Differential Torque to Rotate (DTR) to dimensional shim changes was measured and recorded as part of the rework process. Regression analysis of this data provided factors used to predict of the resultant process uncertainties for DTR,u_{DTR} and backlash,$u_{Backlash}$.

The preload on the bearings is a function of the overall system stiffness coefficient, defined here as k_{Diff}, a direct linear function of the total shim installed. To determine this coefficient, re-shimmed axles were used to generate a data set with total shim dimensional change as the independent variable and change in Differential Torque to Rotate as the dependent variable. Analyzing this data with IBM SPSS Statistics (23.0) Linear Regression, the un-standardized coefficient in the analysis was used as k_{Diff}. Following the guidelines provided in GUM Section 5.2, the values of Pinion Side (PS), Gear Side (GS) are independent, and the covariance term is eliminated in the propagation derivative. Applying this relationship and the uncertainty propagation equation derives a torque to rotate process related to the Differential Torque to Rotate (DTR) uncertainty u_{DTR}.

$$DTR = f\left(k_{Diff}(PS + GS)\right)$$

$$u_{DTR} = \sqrt{k_{Diff}^2(u_{PS}^2 + u_{GS}^2)}$$

The effect on backlash by changes in the Gear Side and Pinion Side shims is more complex. There is an interaction between stiffness and gear position in the assembly resulting from the individual component stiffness. To determine the coefficients, re-shimmed axles were used to generate a data set with Pinion Side shim dimensional change and Gear Side shim dimensional change as the independent variables, and change in backlash as the dependent variable. This data was analyzed with IBM SPSS Statistics (23.0) Linear Regression. The un-standardized coefficients in the analysis were used to determine the backlash response to shim change. Separating the stiffness results for each shim, the backlash effect on Pinion Side factor is k_{PS} and the Gear Side $k_{GS}$was used in the following relation to determine backlash uncertainty $u_{Backlash}$.

$$Backlash = f(k_{PS}PS + k_{GS}GS)$$

$$u_{Backlash} = \sqrt{k_{PS}^2 u_{PS}^2 + k_{GS}^2 u_{GS}^2}$$

Output Measurement Uncertainty Prediction

The final step in developing the model was the application of measurement uncertainties to predict the observed Audit measurement error using standard uncertainties', $u_{TTR-Obs}$ and $u_{Backlash-Obs}$. The audit measurement standard uncertainties derived in Step 1, uncertainty u_{MSTTR} and $u_{MSBacklash}$, were combined with the process uncertainties derived in Step 3 to predict the observed standard uncertainty u_{Obs} in the audit-process. The observed uncertaintyincludes the actual process standard uncertainty u_{Act} combined with the measurement standard uncertainty u_{MS}. Applying standard uncertainty summing techniques, the prediction model for observed backlash and Total Torque to Rotate is expressed in the following equations.

$$u_{Obs}^2 = u_{Act}^2 + u_{MS}^2$$

$$u_{TTR-Obs} = \sqrt{u_{DTR}^2/(ratio)^2 + u_{MSTTR}^2}$$

$$u_{Backlash-Obs} = \sqrt{u_{Backlash}^2 + u_{MSBacklash}^2}$$

Research Design - Question 2

Research Question 2 applied data to the model developed as part of Research Question 1 for analysis. This analysis compares sample measurement data collected from a Supervisory Control and Data Acquisition (SCADA) for the assembly process to a Monte Carlo simulation using measurement uncertainties. This portion of the study included validation of the production data, The Monte Carlo model development, and the statistical comparison of results.

To validate the data, each individual axle was isolated to the first time through the assembly process and duplicates were eliminated using sorting techniques in Excel. Table 4 identifies the measurement variables and characteristics of the measurement data. A graphical image of the measurement variables is included in Appendix A.

Table 4

Independent and dependent variable characteristics

Symbol	Type	Classification	Resolution	Units
CAR1	Independent	Continuous	0.00X	Millimeter (mm)
CAR2	Independent	Continuous	0.00X	Millimeter (mm)
COV	Independent	Continuous	0.00X	Millimeter (mm)
δJ	Independent	Continuous	0.00X	Millimeter (mm)
OAH	Independent	Continuous	0.00X	Millimeter (mm)
BF	Independent	Continuous	0.00X	Millimeter (mm)
PS_{Meas}	Independent	Continuous	0.00X	Millimeter (mm)
GS_{Meas}	Independent	Continuous	0.00X	Millimeter (mm)
Lash	Dependent	Continuous	0.00X	Millimeter (mm)
TTR	Dependent	Continuous	0.0X	Newton-Meter (Nm)

As part of this research the data was reviewed as recommended by Warner (2013, p. 125), reviewing outliers, inconsistencies, and other data anomalies. The data was summarized by individual part unique serial number and a case number assigned as an identifier. The individual measurements were analyzed in IBM SPSS Statistics (23.0) to identify data fliers, assess the distribution, and identify potential special cause variation. Normality was assessed by calculating a z ratio of the excess kurtosis and skewness by their respective standard error (Warner, 2013), thus a z ratio no greater than 2.0 provides a $p \leq .05$ risk of Type 1 error. Datum selected for elimination from the data set was identified along with the rationale for elimination stated in Chapter 4. The final data set for the analysis is summarized and tabulated in Chapter 4.

Monte Carlo Simulation

To generate simulated data, Excel was used to simulate data using the Monte Carlo Method as outlined in the GUM Introduction (JCGM, 2009). The simulation was constructed with cases oriented in rows and the process simulated in columns. An initial study included simulating random true values for each in-process measured variable using the NORM.INV function to produce 5,000 cases of true values aligned in rows. Using the same method, 5,000 measurement error values were simulated based on a normal distribution. The combined uncertainty for each in-process variable applied the uncertainties developed in answering Research Question 1. The MCM simulation calculated an observed value for each variable by adding the measurement error to the true value. The process proceeded for each case producing results for 5,000 unique cases as shown in Figure 11.

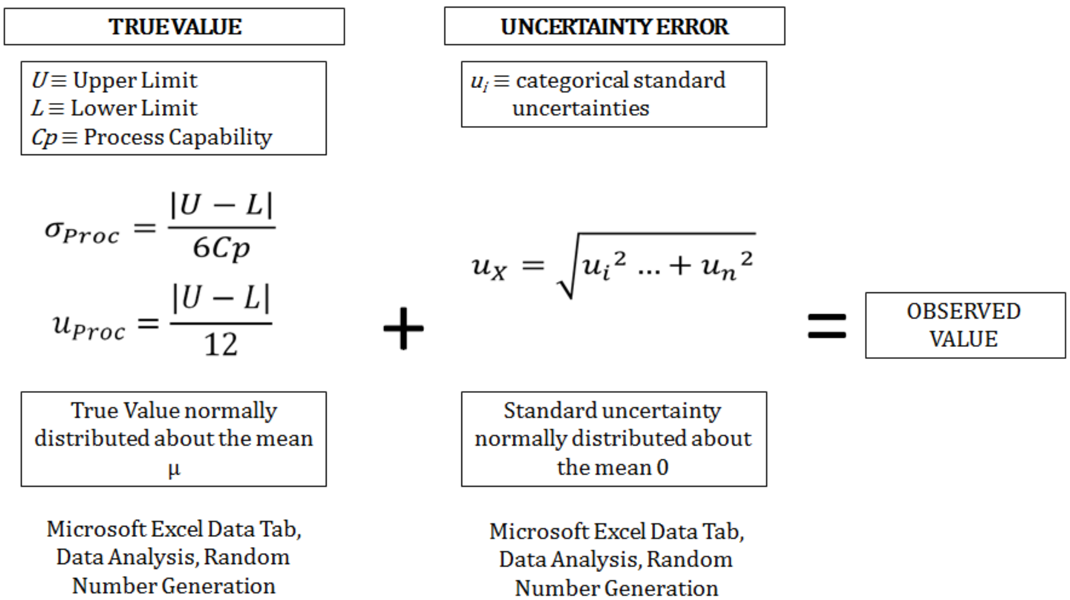

Figure 11. Simulation observed value determined from true value and standard uncertainty

The simulation proceeds on a case-by-case basis following the assembly process. Excel is used to calculate observed and true values for Pinion and Gear Side shims using Equations 1 and 2. The difference between the observed and true values is the shim error for each case. The effect of this error on the process DTR and Backlash is determined by applying the k_{Diff}, k_{PS}, and k_{GS} factors developed in answering Research Question 1. The DTR and Backlash errors are determined on a case-by-case basis with Equations 6 and 7 in the simulation.

$$\epsilon_{DTR} = k_{Diff} * (\epsilon_{GS}) \tag{6}$$

$$\epsilon_{Backlash} = k_{PS} * \epsilon_{PS} + k_{GS} * \epsilon_{GS} \tag{7}$$

The simulation design for generating DTR and backlash process error is shown in Figure 12.

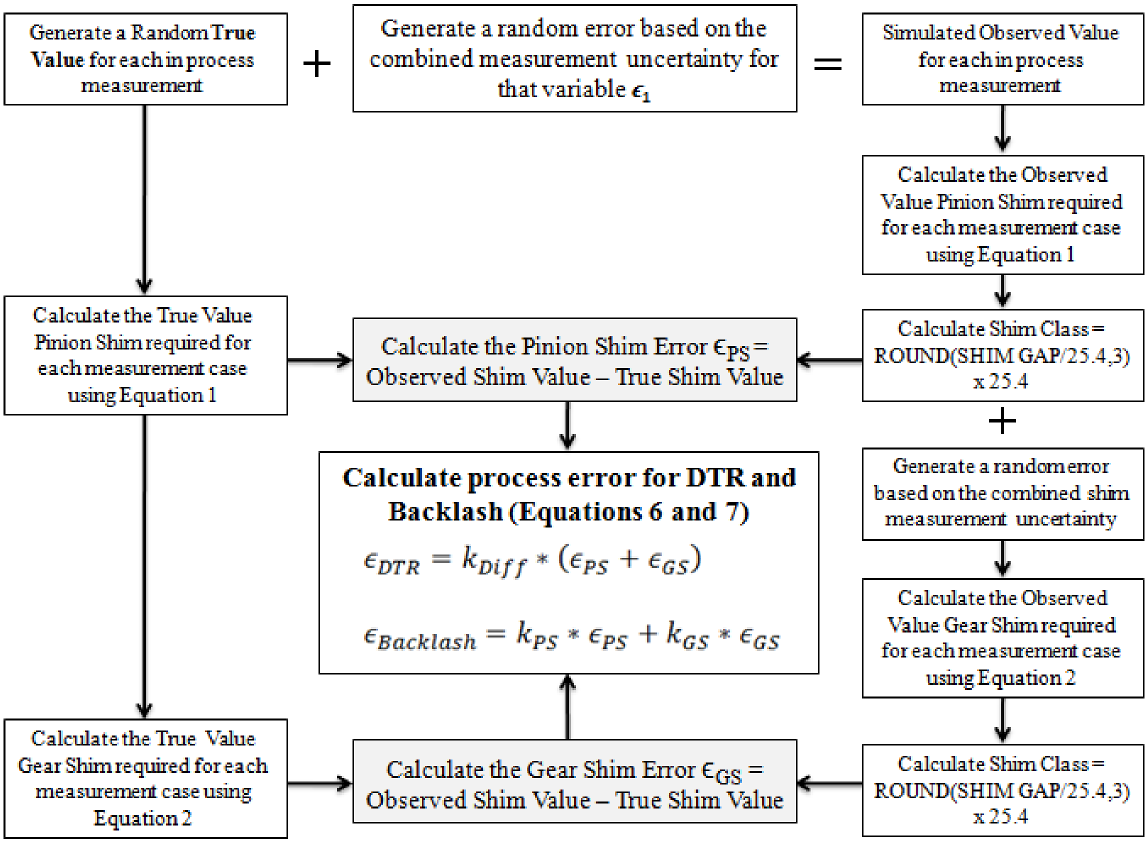

Figure 12 .Simulation calculation of process error flow diagram for DTR and Backlash

The simulated process error for DTR and Backlash produce results comparable to the actual process variance. The simulation proceeds on a case-by-case basis adding the calculated process error to true values to generate input values for DTR and Backlash. The measurement error associated with the audit process is added to this input value to generate a simulated audit result for each case. The process for TTR is shown in Figure 13, the process for Backlash is shown in Figure 14. The results generate the data set that is used for comparison to the collected SCADA data.

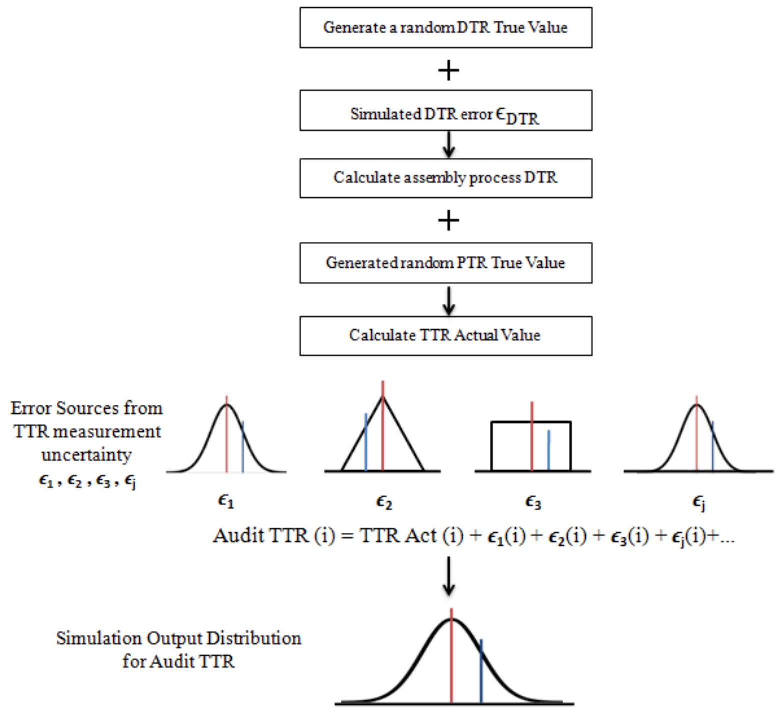

Figure 13. Simulation process for generating Audit TTR values

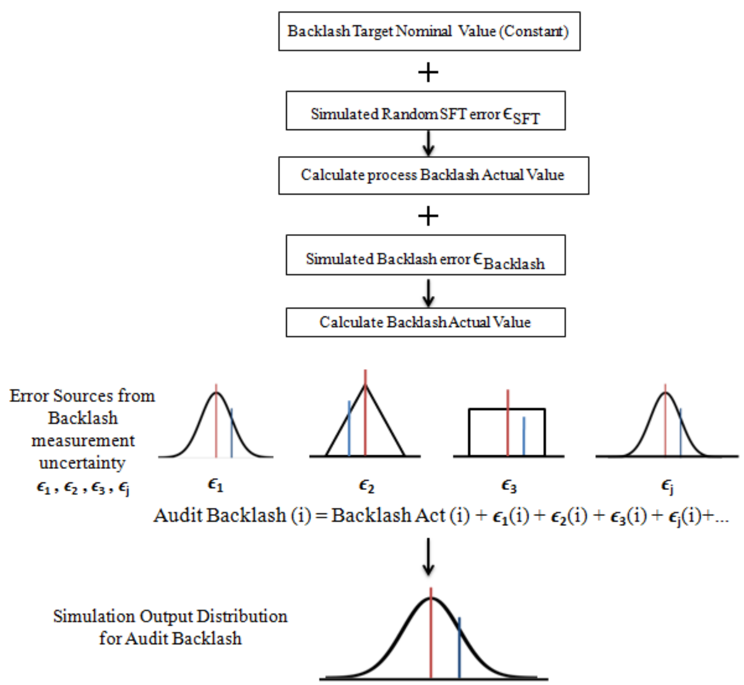

Figure 14. Simulation process for generating Audit Backlash values

The GUM provides guidelines on the number of iterations in the Monte Carlo simulation, where each iteration generates a unique datum. The GUM guidelines for a priori setting of Monte Carlo iterations is at least 10^4 times greater than $1/(1-p)$ where p is the published confidence interval for the supplied uncertainty (JCGM, 2008c). An alternate adaptive approach is included in the GUM (JCGM, 2008c) to continue increasing number of Monte Carlo trials until the parameter of interest has stabilized. The adaptive approach was the method selected for this research. The Monte Carlo trials were selected such that the calculated standard deviation from the simulation was stabilized the equivalent process capability by more than ±0.05. This approach was selected based on the GUM description of the adaptive Monte Carlo stabilization criteria considering the purpose of this research. The technique of monitoring the sample standard deviation for stabilization is consistent with the approach recommended by Coleman and Steele (2009).

Monte Carlo Simulation Validation

To validate the MCS production SCADA data observed results for each measurement was compared to the model prediction. This included an assessment of skewness and kurtosis of the derived actual distribution based on the observed and predicted measurement uncertainty. This assessment provided a first evaluation of the model validity by identifying inconsistencies between the model and the data. Observed values are used as it is not possible to separate the measurement uncertainty from the true value in the SCADA data. The analysis results are part of a tabulated summary of the observed, predicted uncertainty, and estimated actual variation in Chapter 4.

The typical application for Monte Carlo simulation is to determine coverage factors for the resultant measurements or to compare results from other uncertainty frameworks. This study applied the techniques for a different purpose; to determine a distribution of the output variables for comparison to actual data. Complex numerical simulation model validation (e.g. Computational Fluid Dynamic models) as discussed by Coleman and Steele (2009) is not applicable to this study. This study simulation employs basic mathematic expressions where the simulation numerical error is eliminated by validating the simulation calculations on a sample case.

Uncertainty simulation validation methods for this study included a comparison of the simulation predicted output for the audit variables to actual output measurements. The study method follows the guidelines of the GUM Supplement 1, *Guide to the expression of uncertainty in measurement" –propagation of distributions using a Monte Carlo method* (JCGM, 2008c), and Coleman and Steele (2009). The evaluation technique for the effectiveness of the shim-selection uncertainty model compares the simulation distribution and standard deviation to the SCADA data for the audit measurements of Backlash and DTR error. A comparison of the means using independent samples t-test (Warner, 2013)is applied. It was anticipated that both process results would be centered about the nominal mean. This test served to identify any bias in the model or the SCADA data. The risk to the research conclusions due to Type I error in this means comparison is judged very low; a significance of 0.05 was used. If the MCS mean had differed significantly from the target, the simulation could have been iterated to include a bias on one of the input parameters. Assessing the ability of the model to predict variance is the primary goal of this research. To determine the ability of the model to predict the measurement process variance, the F-Test was used to compare variance of the simulation results to the SCADA data. The risk associated with Type I error is considered low. It was the goal of the research to identify if uncertainty may be used to predict a shim-selection measurement system capability.

In many instances, this may be used as a comparison of alternative systems. In those instances, an uncertainty model's ability to predict absolute variance would not be as important as a measure of one alternative performs in comparison to another at reducing process variance. The significance level for the F-Test is p=0.05.

Research Design - Question 3

Research Question 3 seeks to answer the question can the model be used to determine the acceptance limits for an individual in-process shim-selection measurement device. The static housing parameter Cover bearing bore depth (COV) was selected for comparison. The model was used to compare simulation results with a 10% GR&R to results at 50% GR&R using MSA methods based on drawing tolerance. A comparison of the means using independent samples t-test (Warner, 2013)was applied. To determine the ability of the model to predict the measurement process variance, the F-Test was used. Significance levels for both tests is p=.05.

Summary

This chapter identified the quantitative research methods that were used to correlate input measures and resulting audit measures applying uncertainty analysis techniques. The methodology was divided into three stages. The first stage combined uncertainty for each measurement variable and assembled the uncertainties into a model of the system. The second stage includes the uncertainties with simulated true values in a Monte Carlo simulation to generate simulated data of the selected shim selection system. The third stage applied the model to assess the effects of measurement capability for a specific independent variable on the performance of the measurement system.

CHAPTER 4

RESULTS

The purpose of this study was to define and assess a method to predict the performance of an axle shim-selection measurement system. This Chapter describes the study results following the method of investigation described in Chapter 3. To address the research questions this Chapter is divided into three sections that provide results for each of the research questions. The first section presents the derivation of the shim selection uncertainty model. This includes the calculation of each variable uncertainty to address Research Question 1, how can measurement uncertainty methods be applied to model the axle shim-selection measurement process? The section includes the correlation analysis of backlash and DTR response to shim change and derivation of the correlation factors. The first section concludes with the uncertainty results that were used for the model simulation.

The second section addresses Research Question 2, can a measurement system uncertainty model be used to predict the backlash and torque-to-rotate capability of a shim-selection measurement system? This question is addressed by the following hypotheses:

H_{O1}: There is no significant difference between the Means of the uncertainty prediction model and actual test data in Backlash Audit.

H_{A1}: There is significant difference between the Means of the uncertainty prediction model and actual test data in Backlash Audit.

H_{O2}: There is no significant difference between the Variance of the uncertainty prediction model and actual test data in Backlash Audit.

H_{A2}: There is significant difference between the Variance of the uncertainty prediction model and actual test data in Backlash Audit.

H_{O3}: There is no significant difference between the Means of the uncertainty prediction model and actual test data in Audit Total Torque to Rotate.

H_{A3}: There is significant difference between the Means of the uncertainty prediction model and actual test data in Total Torque to Rotate.

H_{O4}: There is no significant difference between the Variance of the uncertainty prediction model and actual test data in Total Torque to Rotate.

H_{A4}: There is significant difference between the Variance of the uncertainty prediction model and actual test data in Total Torque to Rotate.

The third section addresses Research Question 3, can a measurement system uncertainty model be used to determine the acceptance limits for an individual in-process shim-selection measurement apparatus? This question is addressed by the following hypotheses:

H_{O5}: There is no significant difference between the Means of the uncertainty prediction model Audit Backlash with COV measurement capability at 10% and 50%.

H_{A5}: There is significant difference between the Means of the uncertainty prediction model Audit Backlash with COV measurement capability at 10% and 50%.

H_{O6}: There is no significant difference between the Variance of the uncertainty prediction model and Audit Backlash with COV measurement capability at 10% and 50%.

H_{A6}: There is significant difference between the Variance of the uncertainty prediction model Audit Backlash with COV measurement capability at 10% and 50%.

H_{O7}: There is no significant difference between the Means of the uncertainty prediction model Audit Torque-to-Rotate with COV measurement capability at 10% and 50%.

H_{A7}: There is significant difference between the Means of the uncertainty prediction model Audit Torque-to-Rotate with COV measurement capability at 10% and 50%.

H_{O8}: There is no significant difference between the Variance of the uncertainty prediction model Audit Torque-to-Rotate with COV measurement capability at 10% and 50%.

H_{A8}: There is significant difference between the Variance of the uncertainty prediction model and Audit Torque-to-Rotate with COV measurement capability at 10% and 50%.

Research Question 1

Research question one asks, how can measurement uncertainty methods be applied to model the axle shim-selection measurement process?

As described in Chapter 3, eleven components from the ISO (2012) Standard 22514-7 listed in Table 3 were used to evaluate the measurement uncertainty for each variable. This Section calculates and applies each uncertainty component to the specific measurement. The findings divide the uncertainty calculation by the type of measurement, static, dynamic, process, and audit due to the commonalities of each type. For each variable type, a Table that lists the standard uncertainties for the variable follows a discussion of the standard uncertainty rationale.

Two uncertainty components, appraiser reproducibility u_{AV} and appraiser interactions u_{IAi} relate to appraiser contributions to uncertainty. The repeatability study for CAR1, CAR2, COV, and δJ include appraiser reproducibility calculation and the resultant uncertainty is included. The other measurements exclude appraiser uncertainty as not applicable since the other process measurements are automatic. The guidelines provided in ISO (2012) include the greater of resolution uncertainty u_{RE} or repeatability on standards u_{EVR}. In all cases, a consideration of repeatability on a standard master part was included and resolution uncertainty excluded. Measurement GR&R was collected from available sources and included a variety of methods. The method and results for the Type A uncertainties are summarized in Appendix B.

Static measurement independent variables.

Static measurements included as independent variables in this study share uncertainty components. The housing and cover measurement variables CAR1, CAR2, and COV are measured on a precision gauge in similar manner. Gauge reliability and repeatability studies provided an assessment of performance of the measurement apparatus. The other static measurements PS_{Meas} and GS_{Meas} are shim measurements of the selected shim with a common verifying apparatus. The development of standard uncertainty allocations for the static measurements are described below and tabulated in Table 5 for CAR1, Table 6 for CAR2, Table 7 for COV, and Table 8 as a common uncertainty for PS_{Meas} and GS_{Meas}. The uncertainty for each static measurement is combined as a single standard uncertainty value following the method published by ISO (2012).

$$u_{STATIC} = \sqrt{u_{LIN}^2 + u_{BI}^2 + u_{EVR}^2 + u_{EVO}^2 + u_{AV}^2 + u_{STAB}^2 + u_{OBJ}^2 + u_{REST}^2}$$

Two standard uncertainties related to linearity and bias, u_{LIN} and u_{BI} are typically determined experimentally by repeated measures on reference standards as outlined in Section

7.1.2 *Repeatability and bias based on one reference standard*, and 7.1.3 *Linearity analysis based on a minimum of three reference standards* of the ISO (2012)22514-7 standard. Linearity is determined through linear regression comparing the measurement system to a minimum of three standards covering the measurement range. The devices in this study used one mean reference master as the standard, an approach common in shim selection measurement. The standard uncertaintyu_{LIN} is assumed zero for static measurements based on two factors. First, linearity is not significant due to the limited range of each measurement, all are less than 0.200 mm. Second, the utilization of digital probes excludes the effects of any gain related non-linearity.

The ISO (2012) standard includes a method to combine bias u_{BI} and repeatability on standards in the u_{EVR} standard uncertainty, the method selected for this variable. Bias and linearity uncertainty are combined and included in the uncertainty associated with repeatability on the standard. A known set-up master serves as the reference standard and the uncertainty u_{EVR} is categorized as a Type B uncertainty. Assuming a repeatability of ±1 micron, which is typical in the axle manufacturing industry, a calculated rectangular distribution with 0.002 mm range provides a standard uncertainty probability (JCGM, 2008c), $u_{EVR} = 0.002/\sqrt{3} = 0.00115\ mm$. This uncertainty value is applied to all three static gauge measurements CAR1, CAR2, COV, and the shim verifier PS_{Meas} and GS_{Meas}.

Standard uncertainties associated with repeatability and reproducibility (GR&R) studies are Type A uncertainties calculated from repeated measurements. The uncertainties of repeatability on the workpiece u_{EVO}, appraiser reproducibility u_{AV}, and appraiser workpiece interactions u_{IAi} are determined from repeated measures studies. The ANOVA study method published in both the ISO (2012) 22514-7 Standard and the fourth edition of AIAG (2010) MSA was applied to measurements CAR1, CAR2, and COV. The studies are included in Appendix B.

This technique separates the variance contributions of the equipment, appraiser, and interactions. The analysis did not identify any significance for appraiser influence u_{AV}, CAR1 F(2) = 0.51 with p = .60, CAR2 F(2) = 0.44 with p = .64, and COV F(2) = 0.38 with p = .69. Similarly, there is no interaction significance u_{IAi}, CAR1 F(2) = 0.16 with p > .99, CAR2 F(18) = 0.48 with p = .90, and COV F(18) = 1.00 with p = .48. Appraiser and interaction standard uncertainties were excluded, the repeatability on test parts u_{EVO} is included in the respective uncertainty summary table for each variable.

The verifier measurement uncertainties PS_{Meas} and GS_{Meas} are not susceptible to appraiser influence and apply an average range approach classified as a Type 3 Study and described by Dietrich and Schulze (2011). The shim measurement uncertainty u_{EVO} equals the estimated standard deviation determined by the average range of a twenty-five part study with two measurements on each part, $\sigma = \bar{\bar{R}}/d_2^*$. Data from the repeatability studies is included in Appendix B.

Stability uncertainty u_{STAB} of the measurement process is included by both ISO (2012) and Dietrich and Schulze (2011) as an element of the combined uncertainty. Dietrich and Schulze recommended a time based assessment of stability while the ISO standard suggest an extended ANOVA model as a possible approach provided interactions are excluded. At issue is selecting a process that can isolate stability uncertainty from repeatability on the master or a reference part. The shim selection process typically re-masters the gauges periodically to eliminate stability influences. There was consideration for stability associated with the process and this uncertainty was an opportunity to apply a Type B method, which does not rely on statistical studies. The selected method applied a Dietrich and Schulze (2011, pp. 208-209)criteria assuming a normal distribution with two standard deviations equal to one-half the range R_g. For this calculation, a rangeR_g of 0.002 mm is assumed, $u_{STAB} = \sigma = R_g/4 = 0.0005$mm.

Measurand feature variation uncertainty symbolized as u_{OBJ} is what ISO (2012) describes as inhomogeneity. For shim selection static measurements CAR1, CAR2, and COV the locating datum is three-point contact on the faying surface between the Cover and Carrier housings. The three-point contact does not accommodate the datum feature variation of flatness with a drawing tolerance of 0.100 mm. An uncertainty analysis approach for part variation similar to that described by Dietrich and Schulze (2011, pp. 201-204) is applied to the static measurement. This method applies the part tolerance based on the expected process capability of 2.0 C_p and a normal distribution. The uncertainty is equivalent to the process standard deviation, $u_{OBJ} = TOL/(6 * C_p) = 0.05/(6 * 2) = 0.00417\ mm$ included in Table 5, Table 6, and Table 7. In a similar manner the flatness tolerance of the shim is 0.0127 mm, $u_{OBJ} = 0.00106\ mm$, included in Table 8.

To account for uncertainty components not specifically categorized or included in the process ISO (2012)creates a category u_{REST}. For the static measurements, an uncertainty associated with the digital measurement probe is included in this classification. The probe manufacturers published repeatability is 0.00015 mm, assuming a rectangular distribution the uncertainty is calculated, $u_{REST} = 0.00015/\sqrt{3} = 0.00009\ MM$. This value is not significant relative to other factors but it is included for reference.

As stated earlier, the following Table 5 through Table 8 summarizes the uncertainty contribution from each independent static variable. The last row in each table provides the resultant uncertainty for that variable which is included in the uncertainty analysis and the Monte Carlo simulation.

Table 5

Standard uncertainty allocation for variable CAR1

Symbol	*U* Source	*u* Type	*u* Value	Units
u_{EVR}	Repeatability on standard (master)	Type B	0.00115	mm
u_{EVO}	Repeatability on workpiece	Type A	0.00266	mm
u_{AV}	Appraiser reproducibility	Type A	0.00000	mm
u_{IAi}	Interactions	Type A	0.00000	mm
u_{STAB}	Reproducibility over time	Type B	0.00050	mm
u_{OBJ}	Measurand non-homogeneity	Type B	0.00417	mm
u_{REST}	Other uncertainty contributors	Type B	0.00009	mm
u_{CAR1}	Combined uncertainty		0.00510	mm

Table 6

Standard uncertainty allocation for variable CAR2

Symbol	*u* Source	*u* Type	*u* Value	Units
u_{EVR}	Repeatability on standard (master)	Type B	0.00115	mm
u_{EVO}	Repeatability on workpiece	Type A	0.00045	mm
u_{AV}	Appraiser reproducibility	Type A	0.00000	mm
u_{IAi}	Interactions	Type A	0.00000	mm

Symbol	u Source	u Type	u Value	Units
u_{STAB}	Reproducibility over time	Type B	0.00050	mm
u_{OBJ}	Measurand non-homogeneity	Type A	0.00417	mm
u_{REST}	Other uncertainty contributors	Type B	0.00009	mm
u_{CAR2}	Combined uncertainty		0.00438	mm

Table 7

Standard uncertainty allocation for variable COV

Symbol	u Source	u Type	u Value	Units
u_{EVR}	Repeatability on standard (master)	Type B	0.00115	mm
u_{EVO}	Repeatability on workpiece	Type A	0.00054	mm
u_{AV}	Appraiser reproducibility	Type A	0.00000	mm
u_{IAi}	Interactions	Type A	0.00000	mm
u_{STAB}	Reproducibility over time	Type B	0.00050	mm
u_{OBJ}	Measurand non-homogeneity	Type A	0.00417	mm
u_{REST}	Other uncertainty contributors	Type B	0.00009	mm
u_{COV}	Combined uncertainty		0.00439	mm

Table 8

Standard uncertainty allocation for variables PS_{Meas} and GS_{Meas}

Symbol	u Source	u Type	u Value	Units
u_{EVR}	Repeatability on standard (master)	Type B	0.00115	mm
u_{EVO}	Repeatability on workpiece	Type A	0.00328	mm
u_{STAB}	Reproducibility over time	Type B	0.00050	mm
u_{OBJ}	Measurand non-homogeneity	Type A	0.00106	mm
u_{REST}	Other uncertainty contributors	Type B	0.00009	mm
$u_{Shim\ Meas}$	Combined uncertainty		0.00367	mm

Dynamic Measurement Independent Variables

The independent variables classified as dynamic measurements are measured while the test article is rotated. These variables share common uncertainties associated with bearing and gear variability. Three variables are included in this category, OAH, BF, and δJ. Gauge repeatability studies provide an assessment of performance of the measurement apparatus and are included in Appendix B. The standard uncertainties related to linearity and bias, u_{LIN} and u_{BI} are typically determined experimentally with linear regression methods by repeated measures on reference standards as outlined in the ISO (2012) 22514-7 standard. Similar to static measurements, dynamic measurements include one mean master as the standard. Linearity is not significant due to the limited range of each measurement, all are less than 0.200 mm. Further, the utilization of digital probes excludes the effects of any gain related non-linearity. This study analysis followed the ISO (2012) standard method of combining bias u_{BI} and repeatability on masters with u_{EVR} standard uncertainty. A known set-up master serves as the reference standard and the uncertainty u_{EVR} is categorized as a Type B uncertainty. The uncertainty for each dynamic measurement was combined as a single standard uncertainty value following the published ISO (2012) method.

$$u_{DYNAMIC} = \sqrt{u_{LIN}^2 + u_{BI}^2 + u_{EVR}^2 + u_{EVO}^2 + u_{AV}^2 + u_{STAB}^2 + u_{OBJ}^2 + u_{REST}^2}$$

Assessing the uncertainty u_{EVR} of the dynamic measurement process on a standard includes a compromise in that the standard measurements for OAH and BF do not include dynamic rotation. The nature of the uncertainty fits the *GUM 100:2008 Section 4.3*(JCGM, 2008a) description of a Type B uncertainty, one that involves insight based on experience and general knowledge. Two factors were considered in establishing u_{EVR} for the dynamic measurements. The first is the impracticality of applying a dynamic master; static set-up masters are employed in all measurement applications. The second is the nature of the shim selection process, the application of shim offsets make the process robust to bias associated with variability on the master. To allocate this uncertainty, the value of the Master flatness variation of ±0.0015 mm is included to account for varying positioning of the Master. A calculated rectangular distribution with 0.003 mm range provides a standard uncertainty (JCGM, 2008c), $u_{EVR} = 0.003/\sqrt{3} = 0.00173\ MM$. This standard uncertainty value is applied to the dynamic measurements OAH and BF.

A repeatability study on a standard was conducted for δJ allowing a Type A u_{EVR} calculation. The uncertainty was calculated as the standard error based on repeated measures on a standard gearset per ISO (2012) Statistical Methods Table 4, Instance 1. The data for calculation of δJ u_{EVR}applied 33 measurements to calculate $u_{EVR} = .00214$is included in Appendix B.

Uncertainties associated with repeatability and reproducibility (GR&R) studies were determined by Type A methods using repeated measurements. The uncertainties of repeatability on the workpiece u_{EVO} for the independent variables OAH and BF applied the Average and Range method in a Type 3 study (Dietrich & Schulze, 2011). The independent variable δJ applied the ANOVA study method published in both the ISO (2012) 22514-7 Standard and the Fourth Edition of AIAG (2010) MSA to assess appraiser influence. The appraiser reproducibility u_{AV} was significant F(2) = 11692 with p <.001, and appraiser workpiece interactions u_{IAi} was not, F(18) = 0.59 with p = .90. The resultant u_{EVO} for each variable is summarized in Table 9 through Table 11.

Stability uncertainty u_{STAB} of the measurement process is included by both ISO (2012) and Dietrich and Schulze (2011) as an element of the combined uncertainty. The same approach described for static measurements is applied. The consideration for stability uncertainty applies a Type B method not relying on statistical methods. The selected method applies Dietrich and Schulze (2011, pp. 208-209) assuming a normal distribution with two standard deviations equal to one-half the range R_g. For this calculation, an estimate is based on the repeatability of individual studies where a range of 0.002 mm is typical, consequently$u_{STAB} = \sigma = R_g/4 = 0.002/4 = 0.0005$mm.

Measurand inhomogeneity or feature variation uncertainty u_{OBJ} applies differently to each dynamic measurement. The dynamic measurement variable OAH is measured and averaged over several part rotations and the bearings are precision manufactured. This process accounts for inhomogeneity, $u_{OBJ} = 0.00$ for OAH. The dynamic variable δJ is determined in a

Single Flank test machine with the gears rotated over several revolutions, giving$u_{OBJ} = 0.00$ for δJ. The inhomogeneity in the dynamic variable BF is a measurement recorded and averaged, including a measurement of the variation of the BF measurement. That is the difference between the max and minimum readings characterized as run-out of the BF parameter. Repeatability studies using the Average and Range method in a Type 3 study (Dietrich & Schulze, 2011) of the run-out parameter are included in Appendix B, the standard deviation calculated from the repeatability studies is applied as the standard uncertainty for the BF u_{OBJ}.

The category of other uncertainty for OAH and BF follows the approach described for static measurements. The digital measurement probe published repeatability is 0.00015 mm, assuming a rectangular distribution the uncertainty is $u_{REST} = 0.00009\ mm$. This value was not significant but is included for reference. There is an uncertainty consideration for δJ relative to the position of the pinion in the single flank tester. The allowable variation of the pinion mounting position in the tester is ±0.05 mm. There is a known relationship of pinion position variation to backlash of 0.24(Mohsen Kolivand PhD - Manager AAM Gear Design and Research, personal communication, November 23, 2016). Applying a Type B method to account for this uncertainty where the 0.10 mm range includes 90% probability normal distribution, $z = \pm 1.645$, $u_{REST\delta J} = 0.10/(2 * 1.645) = 0.00729$ mm(Dietrich & Schulze, 2011).

The following Tables 9 through 11 summarize the uncertainty contribution from each dynamic measurement independent variable. The last row in each table provides the resultant uncertainty for that variable which is included in the uncertainty analysis and the Monte Carlo simulation.

Table 9

Standard uncertainty allocation for variable OAH

Symbol	*u* Source	*u* Type	*u* Value	Units
u_{EVR}	Repeatability on standard (master)	Type B	0.00173	mm
u_{EVO}	Repeatability on workpiece	Type A	0.00762	mm
u_{STAB}	Reproducibility over time	Type B	0.00050	mm
u_{OBJ}	Measurand non-homogeneity	Type B	0.00000	mm
u_{REST}	Other uncertainty contributors	Type B	0.00009	mm
u_{OAH}	Combined uncertainty		0.00783	mm

Table 10

Standard uncertainty allocation for variable BF

Symbol	*u* Source	*u* Type	*u* Value	Units
u_{EVR}	Repeatability on standard (master)	Type B	0.00173	mm
u_{EVO}	Repeatability on workpiece	Type A	0.00269	mm
u_{STAB}	Reproducibility over time	Type B	0.00050	mm
u_{OBJ}	Measurand non-homogeneity	Type A	0.00046	mm
u_{REST}	Other uncertainty contributors	Type B	0.00009	mm
u_{BF}	Combined uncertainty		0.00328	mm

Table 11

Standard uncertainty allocation for variable δJ

Symbol	u Source	u Type	u Value	Units
u_{EVR}	Repeatability on standard (Master)	Type A	0.00214	mm
u_{EVO}	Repeatability on workpiece	Type A	0.00189	mm
u_{AV}	Reproducibility of Appraiser	Type A	0.00162	mm
u_{STAB}	Reproducibility over time	Type B	0.00050	mm
u_{OBJ}	Measurand non-homogeneity	N/A	0.00000	mm
u_{REST}	Other uncertainty contributors	Type B	0.00729	mm
$u_{\delta J}$	Combined uncertainty		0.00801	mm

Shim Selection Uncertainty Model

To determine the uncertainty of the shim selection process, the independent variables combine with an uncertainty associated with the shim class selection for the process uncertainty. Shims are classified in 0.0254 mm (0.001 inch) increments. There is an uncertainty u_{STEP} associated with the shim selected based on the classification requirement and the drawing tolerance ±0.013 mm. The shim selection is analogous to resolution uncertainty per ISO (2012)*Table 2 - Uncertainty from resolution*, the shim selection increment is a rectangular distribution. The uncertainty associated with shim class was calculated based on a rectangular distribution of one-half the shim class,

$u_{CLASS} = (1/\sqrt{3}) * (Shim\ Step/2) = Shim\ Step/\sqrt{12} = 0.0254/\sqrt{12} = 0.00733\ mm.$

Standard uncertainty associated with tolerance applies the approach for part tolerance similar to that described by Dietrich and Schulze (2011, pp. 201-204). The shim manufacturing process does not control to specific tolaerance. The process manufacturers shims and then sorts them into classes. To allocate uncertainty a 1.0 C_p and a normal distribution is applied. The uncertainty is then equivalent to the process standard deviation, $u_{STEP} = TOL/6 * C_p = 0.026/(6 * 1) = 0.00433\ mm$.

The uncertainty analyses for the individual elements are combined following the GUM guidelines using the methods described in Chapter 3. The combined standard uncertainty for the Pinion Side shim selection u_{PS} is summarized in Table 12, the last row is the uncertainty calculated using Equation 4, $u_{PS} = \sqrt{u_{CAR1}^2 + u_{CAR2}^2 + u_{OAH}^2 + u_{BF}^2 + u_{\delta J}^2 + u_{Step}^2}$. The uncertainty propagation for backlash summarized in Table 13 excludes the variable uncertainty included the Pinion Side calculation, $u_{GS-LASH} = \sqrt{u_{COV1}^2 + u_{PSMeas}^2 + u_{Step}^2}$. The uncertainty propagation for DTR is separate as the Gear Side shim calculation effecting DTR is independent of the Pinion Side calculations. The corresponding uncertainty associated with the Gear Side shim selection is summarized in Table 14, where the last row is the uncertainty calculated using Equation 5, $u_{GS-DTR} = \sqrt{u_{CAR1}^2 + u_{COV1}^2 + u_{PSMeas}^2 + u_{OAH}^2 + u_{Step}^2}$.

Table 12

Standard uncertainty summary for Pinion Side Shim Selection

Symbol	*u* Source	*Source*	*u* Value	Units
u_{CAR1}	Carrier Measure CAR1	Table 5	0.00510	mm

Symbol	*u* Source	*Source*	*u* Value	Units
u_{CAR2}	Carrier Measure CAR2	Table 6	0.00438	mm
u_{OAH}	Dynamic Differential OAH	Table 9	0.00783	mm
u_{BF}	Dynamic Differential BF	Table 10	0.00328	mm
$u_{\delta J}$	Dynamic Gearset δJ	Table 11	0.00801	mm
u_{CLASS}	Shim Class – Pinion Side		0.00733	mm
u_{TOL}	Shim Tolerance – Pinion Side		0.00433	mm
u_{PS}	Pinion Side Shim Uncertainty		0.01594	mm

Table 13

Standard uncertainty summary for Gear Side Shim Selection Backlash

Symbol	*u* Source	*Source*	*u* Value	Units
u_{COV}	Cover Measure COV	Table 7	0.00439	mm
$u_{PS\ MEAS}$	Pinion Side Measurement	Table 8	0.00367	mm
u_{CLASS}	Shim Class – Gear Side		0.00733	mm
u_{TOL}	Shim Tolerance – Gear Side		0.00433	mm
$u_{GS-LASH}$	Gear Side Shim Uncertainty (Backlash)		0.01026	mm

Table 14

Standard uncertainty summary for Gear Side Shim Selection DTR

Symbol	*u* Source	*Source*	*u* Value	Units
u_{CAR1}	Carrier Measure CAR1	Table 5	0.00510	mm
u_{COV}	Cover Measure COV	Table 7	0.00439	mm
u_{OAH}	Dynamic Differential OAH	Table 9	0.00783	mm
$u_{PS\ MEAS}$	Pinion Side Measurement	Table 8	0.00367	mm
u_{CLASS}	Shim Class – Gear Side		0.00733	mm
u_{TOL}	Shim Tolerance – Gear Side		0.00433	mm
u_{GS-DTR}	Gear Side Shim Uncertainty DTR		0.01388	mm

Shim Uncertainty Correlation

The influence of shim standard uncertainty on the dependent variables of Differential Torque to Rotate(DTR) and Backlash includes interaction of bearing preload force and backlash due to relative stiffness of the assembled components. This relationship was determined empirically through post processing data from re-shimmed axles. An empirical model was derived through analysis of the response of Backlash and Total Torque to Rotate (TTR) to shim changes. The response of both dependent variables is as expected due to the linear characteristics of the components. The static components are all in the linear elastic range of materials, bearing torque responds linearly to load (Timken, 2011), and backlash response is linear (Stadtfeld, 2014).

Data collected from forty-three reworked axles were studied to determine the effects of shim change on the two dependent variables, Backlash and DTR. Each axle assembly was audited measuring backlash and DTR and rejected as outside the tolerance limits for one of the two parameters. Subsequently, the Pinion-side and Gear-side shims were changed and the axle reprocessed through the assembly system. The shims for the first time through and after change are measured along with the assembly Backlash and TTR for both configurations. This provides a method to determine the response to the variable of the change in shim. The shim change is the control variable while all other variables are unchanged. This data, included in Appendix C, enables analysis of the effects of independent variable shim delta on the dependent variables.

An evaluation of the effects of change in the total shim was calculated by summing Pinion-side and Gear-side shims. The sum variable influence on the dependent variable DTR was analyzed using SPSS Version 23 linear regression. To provide data consistent between ratios, the differential torque was calculated by multiplying the measured torque on the pinion by the ratio. Linear regression analysis of the data confirmed the relationship between DTR and total shim is linear, $F(41) = 382.3$, ($p<.001$). The model calculated coefficients result in the equation; $Diff\ TTR = 11.83 * TOT\ SHIM\delta - .008$, with the regression model explaining greater than 90% of the variance, R^2=.901. The coefficient 11.83 is significant, $t = 19.55$ (p <.001), while the constant is not, $t = -.251$ (p >.80). The intercept non-significance was expected, the intercept theoretically is zero. The effects of shim change on DTR are dependent on incremental change and are affected only by the slope coefficient. The linear nature of the data and model is shown in Figure 15.

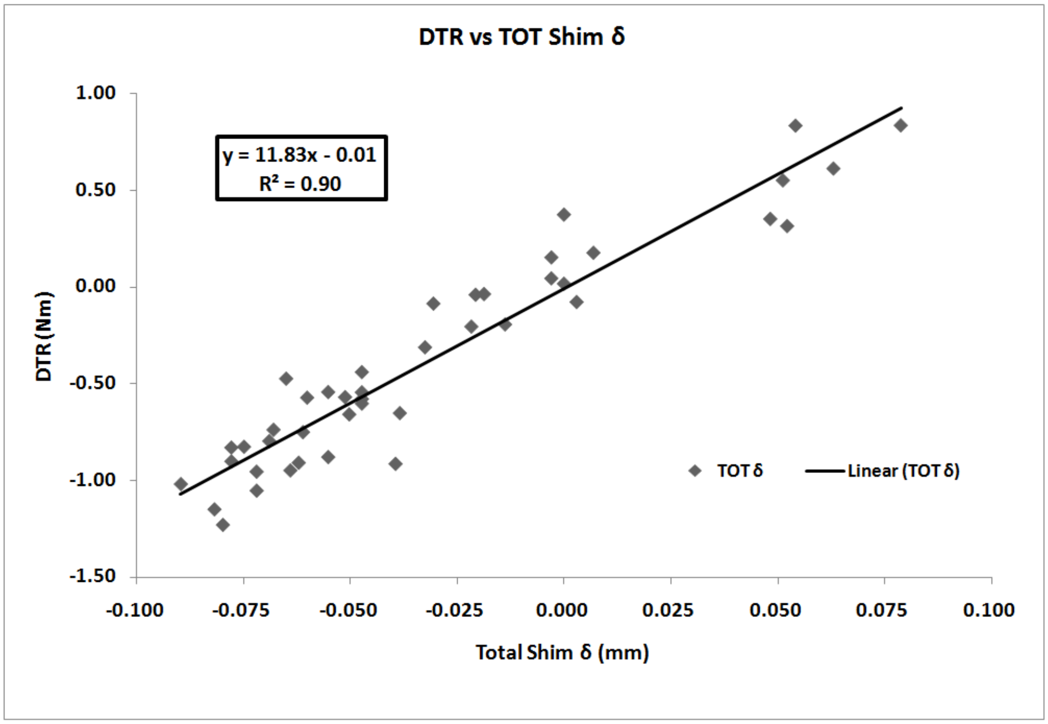

Figure 15. Linear model DTR responses to shim size change

Following the guidelines provided in GUM Section 5.2, the values of Pinion Side (PS), Gear Side (GS) are independent, and the covariance term is eliminated in the propagation derivative. The response of Differential Torque is linear, $DTR = f\left(k_{Diff}(PS + GS)\right) = 11.83 * (PS + GS)$. The process selects the Gear Side Shim to provide differential bearing preload by including the measured Pinion Side Shim such that the error contributing to DTR is part of the Gear Side Shim uncertainty. Applying this relationship and the uncertainty propagation equation derives a differential torque to rotate process uncertainty u_{TTR} , $u_{TTR}^2 = 11.83^2(u_{GS-DTR}^2)$. This value was used for the uncertainty framework calculation and the Monte Carlo Method (MCM) simulation.

The effects of shim changes on the dependent variable Backlash was analyzed by SPSS multiple linear regression methods. The analysis included Pinion-side and Gear-side shim change as independent variables and Backlash as the dependent variable. Linear regression analysis of the data confirmed the relationship between Backlash and shim changes was linear, $F(40)$ = 404.7, (p<.001). The model result; $Backlash = (-0.489) * GS\delta + 0.474 * PS\delta - 0.001$ explained greater than 95% of the variance, R^2=0.95. The Gear-side coefficient -.489 is significant, t = -12.80 (p <.001), the Pinion-side coefficient .474 is significant, t = 12.25 (p <.001), while the constant is not, t = -0.71 (p >.48). The intercept non-significance is expected, the intercept theoretically is zero. The backlash model prediction is compared to actual data in Figure 16.

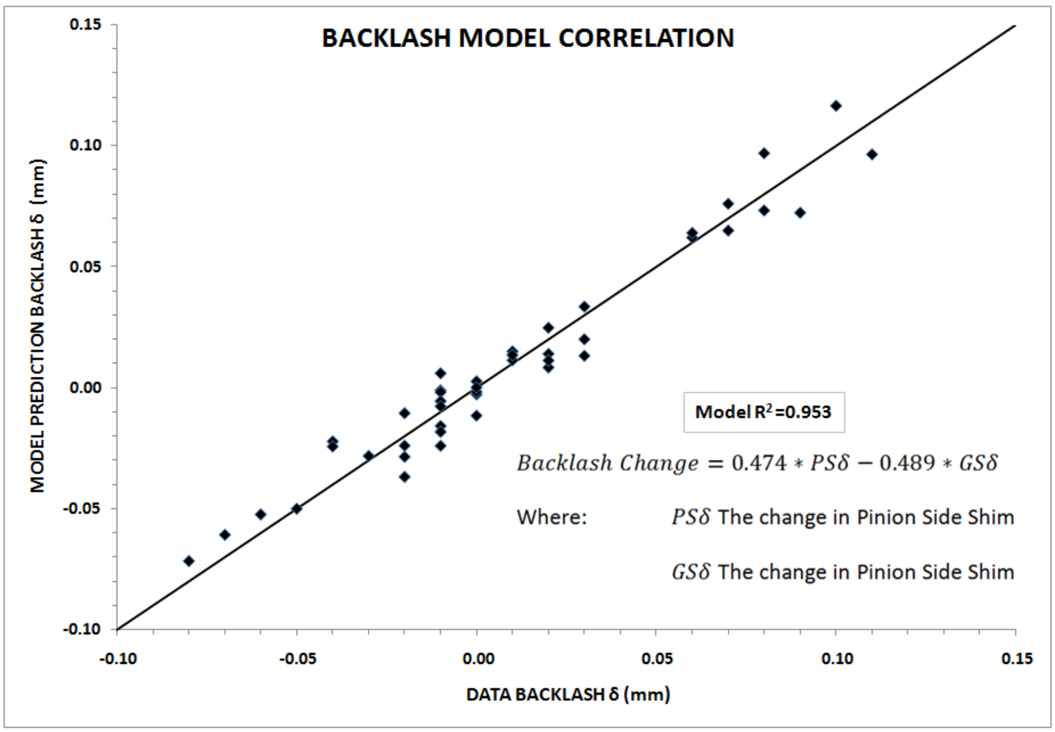

Figure 16. Model backlash prediction to shim change correlation

Following the guidelines provided in GUM Section 5.2, the values of Pinion Side (PS), Gear Side (GS) are independent eliminating the covariance term in the propagation derivative. The response of Backlash is linear, $Backlash = f(k_{PS}GS + k_{GS}PS) = (-0.489) * GS + 0.474 * PS$. Applying this relationship and the uncertainty propagation equation derives a backlash process uncertainty $u_{Backlash}$, $u_{Backlash}^2 = (-0.489)^2 u_{GS-LASH}^2 + (0.474)^2 u_{PS}^2$. The results of this were applied in the uncertainty framework and MCM simulation.

Audit Measurement Uncertainty

As stated above, the dependent variables in this study are backlash and differential torque to rotate. The measurement apparatus is fully automatic, and so is not susceptible to appraiser or interaction uncertainties. This obviated consideration of appraiser uncertainty u_{AV} and interactions with appraiser uncertainty u_{IAi} as part of gauge repeatability studies assessing uncertainty. The audit measurement apparatus includes high-resolution devices for backlash and torque measurement eliminating resolution uncertainty u_{RE} as a contributor to the audit uncertainty. The measurement devices' high stability removed consideration of time based stability uncertainty u_{STAB} associated with measurement drift and so was not included in the combined audit uncertainty.

The application of standard uncertainties related to linearity and bias, u_{LIN} and $u_{BI}$is established by repeated measures on reference standards as outlined in the ISO (2012) 22514-7 standard but were not applicable to the process. First is the consideration of the limited range of each measurement, backlash less than 0.200 mm, and torque range of less than 2.0 Nm. This combined with the utilization of high-resolution measurement devices eliminate linearity and resolution as discernable uncertainty contributors. As a result, u_{LIN} and u_{RE} were modeled at zero in these instances.

The primary audit measurement system assessment method was repeatability studies used to establish u_{EVO}. The uncertainty from repeatability on test partsu_{EVO} equals the estimated standard deviation determined by Average and Range method in a Type 3 study (Dietrich & Schulze, 2011). The average range applied five measurements on each part to mitigate the influence of part variability. Data from the repeatability studies is included in Appendix B, the results are summarized in Table 16 and Table 17.

The standard uncertainty related to location of measurement in the process u_{GV} has application in torque measurement. The assembly system identifies the various steps in the assembly process as operations shown previously in Figure 3. Each process step is identified as an Operation (OP) and sequential number. For the torque process, the first step is Operation 90 (OP90) that measures the Pinion Torque to Rotate (PTR). The second is Operation 120 (OP120), which measures Differential Torque to Rotate (DTR), the same operation that measures OAH and BF. The final step in the torque measurement process is the Audit operation, Operation 180 (OP180), which measures Total Torque to Rotate (TTR). The process for torque measurement is shown in Figure 17. During the OP120 process, a measured torque for each part becomes the unique process target for that assembly. Limits for the audit measurement on each individual axle are set as ±0.58 Nm about that process target. The following example calculation where the gear ratio is 3.727 corresponds to the values shown in Figure 17.

$$DTR\ Audit = (TTR - PTR) * Gear\ Ratio$$

$$DTR\ Audit\ Error = (DTR\ Audit) - (DTR\ from\ OP120)$$

Calculation example:

$$DTR\ Audit = (2.163 - 1.561) * 3.727 = 2.244\text{Nm}$$

$$DTR\ Audit\ Error = (2.244 - 2.566) = -0.322\text{Nm}$$

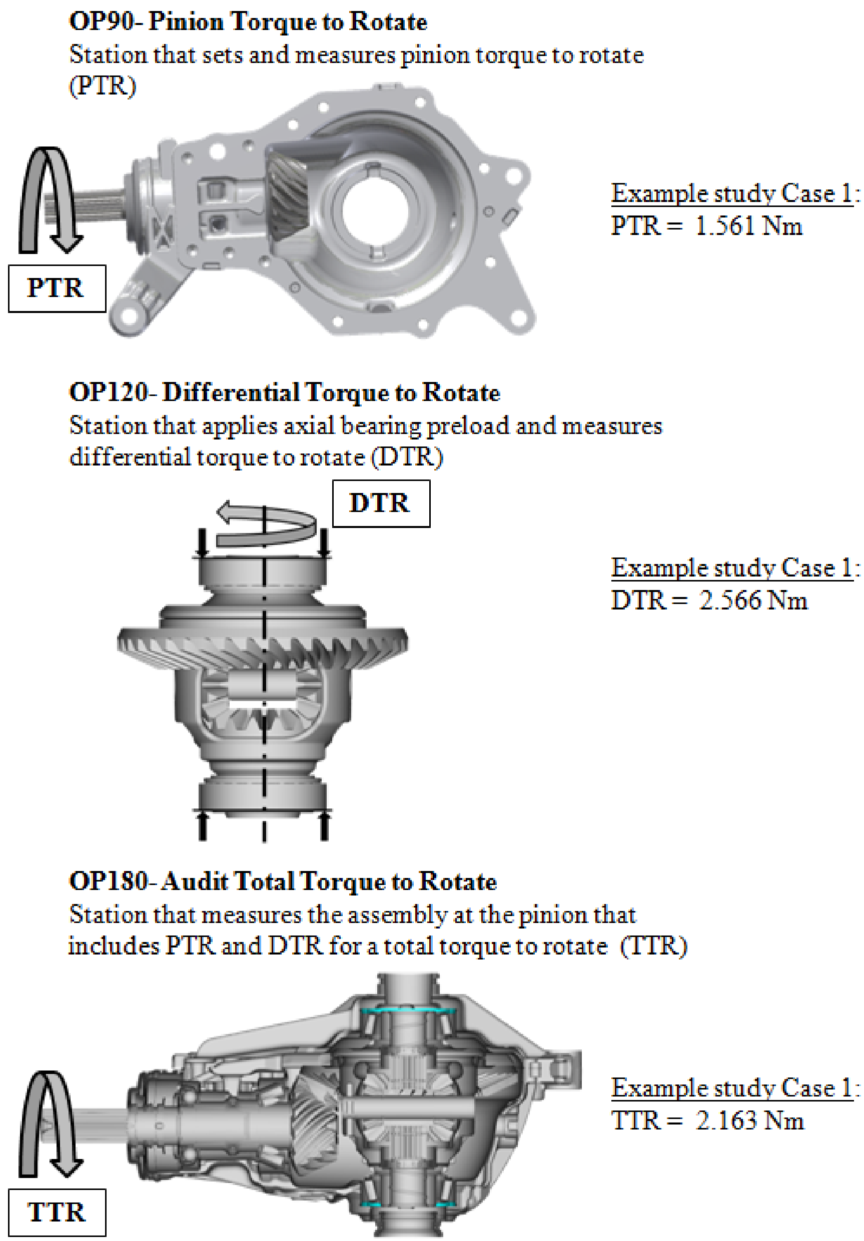

Figure 17. Torque measurement process for determining audit measurement of DTR

As previously stated the upper and lower limits for differential bearing torque are established for each part based on differential torque to rotate (DTR) that is measured in Operation 120. The assembled product does not permit isolation of this torque for the audit measurement of DTR in Operation 180. The audit process measures torque at the Pinion on the assembled axle. This torque includes the pinion bearing and differential bearing torque, referred to as total torque to rotate (TTR) in the audit process. Hence, that measurement includes the pinion torque to rotate (PTR), also measured on each part in the process. The differential bearing torque to rotate is calculated as, $DTR = (TTR - PTR) * Gear\ Ratio$. Rearranging to permit evaluation uncertainty u_{GV}, $TTR = DTR/Ratio + PTR$. Applying GUM (JCGM, 2008a) uncertainty propagation method the resultant uncertainty on the audit measurement was derived,$u_{GV-TTR}^2 = u_{DTR}^2/(Ratio)^2 + u_{PTR}^2$.

Determining the process uncertainty contribution of interim uncertainties $u_{PTR}\ and\ u_{DTR}$included two elements. The repeatability on workpieceu_{EVO} applied Type A uncertainty methods with repeatability studies by the Average and Range method in a Type 3 study (Dietrich & Schulze, 2011) that are included in Appendix B. There is a known uncertainty associated with stability of the DTR and PTR that was included as a standard uncertainty u_{STAB}. This part stability uncertainty is known to be significant and is difficult to quantify. It is related to the bearing torque sensitivity to the rust preventative that is applied to the bearings during bearing manufacture (Johns, Kamping, Krueger, Mynderse, & Riedel, 2016). For this study a Type A approach of reviewing three repeatability studies of bearing torque variation on similar bearings was used. The average standard deviation from the studies indicated a standard deviation in torque of 0.025 Nm per tapered roller bearing. With this criteria the PTR $u_{STAB} = 0.025$ and DTRP $u_{STAB} = 0.05$. The standard uncertainty of u_{DTR} and u_{PTR} and the effect on the audit measurement is summarized in Table 15.

Table 15

Standard uncertainty summary for in-process measurements PTR and DTR

Symbol	*u* Source	*u* Type	u_{PTR}	u_{DTR}	Units
u_{EVO}	Repeatability on workpiece	Type A	0.0199	0.0212	Nm
u_{STAB}	Part Stability	Type A	0.0250	0.0500	Nm
$u_{combined}$	$\sqrt{u_{EVO}^2 + u_{STAB}^2 + u_{OBJ}^2}$		0.0320	0.054	Nm
		RATIO	3.23 (42/13)		
u_{GV-TTR}	$\sqrt{u_{DTR}^2/(ratio)^2 + u_{PTR}^2}$		0.0361		Nm

The standard uncertainty of part inhomogeneity for backlash is associated with the hunting tooth gear geometry requiring a large number of gear rotations to measure all possible tooth combinations (Kish, 1997). The backlash is measured over one revolution of the ring gear resulting in less than 10% of the possible combinations. A Type B method is applied to include the uncertainty of backlash due to the hunting tooth design. The total number of mesh possibilities is the product of the pinion and gear tooth count is, $13 * 42 = 546$ for the 3.23 ratio, one gear revolution includes 42 of the total mesh possibilities. Applying the standard error theory that the sample mean is normally distributed about the true mean including all mesh possibilities, the standard error may be approximated as published by Miller and Freund (1977) as, $\sigma/\sqrt{n}$. Applying the gear manufacturing drawing tolerance of ±0.025 mm rectangular distribution as outlined by Dietrich and Schulze (2011)the standard deviation is estimated as $0.05/\sqrt{3} = 0.0289$mm.The audit sample distribution standard error is used to estimate the standard uncertainty, $u_{OBJ-LASH} = 0.0289/\sqrt{42} = 0.00445$.

This study analysis follows the ISO (2012) method of combining bias u_{BI} and repeatability on masters in the u_{EVR} standard uncertainty. The audit measurements of backlash and TTR are both dynamic and do not apply a standard or master. The process validation for backlash compares equipment readings to manual measurements. The process validation for TTR is a comparison of equipment readings to a reference measurement device. To account for this uncertainty a Type A torque audit u_{EVR} was estimated using a fifteen-part correlation of OP180 Total Torque measurement and the reference torque measurement device. The uncertainty was determined by a regression error method adapted from that described in *Section A.1.3* of the ISO (2012) 22514-7 Standard. Analyzing the sum of squares residual from the correlation study a variance can be determined. The standard deviation calculated from this variance was the value for TTR $u_{EVR} = 0.0266$. The data and results are included in Appendix B Table B48, and Table B49.

The axle audit process includes two backlash uncertainties u_{REST} identified in the ISO (2012) as a category of other uncertainties specific to the measurement process under analysis. The first is an uncertainty associated with variation in the pinion position. A separate uncertainty study is provided in Appendix E, based on that study the resultant standard uncertainty of the pinion position in the assembly, $u_{Pinion-P} = 0.0101\ mm$. Applying the 0.24 relationship of P variation to backlash the$u_{REST1-LASH} = 0.0101 * 0.24 = 0.0024\ mm$.

The secondu_{REST} is a result of the gear manufacturing process that includes a Single Flank Test (SFT) process that does not measure δJ at the nominal normal backlash value of 0.18 mm, but offsets to accommodate total accumulated pitch variation and pitch line run-out (Smith, 1985). The gear drawing indicates a value ranging from 0.13 to 0.18 mm for normal backlash during SFT testing. Applying a Type B method to account for this uncertainty where the 0.05

mm range includes 90% probability normal distribution, $z = \pm 1.645$, $u_{REST2-LASH} = 0.05/(2 * 1.645) = 0.0152\ mm$(Dietrich & Schulze, 2011).

The standard uncertainty allocations for the audit dependent variable measurements are summarized below and tabulated in Table 16 backlash and Table 17 differential torque to rotate (DTR). The uncertainty for each audit measurement is combined as a single standard uncertainty value following the published ISO (2012)method as, $u_{Audit} = \sqrt{u_{EVO}^2 + u_{GV}^2 + u_{OBJ}^2 + u_{REST}^2}$.

Table 16

Standard uncertainty summary for backlash audit measurement

Symbol	*u* Source	*u* Type	*u* Value	Units
u_{EVO}	Repeatability on workpiece	Type A	0.00933	mm
u_{OBJ}	Measurand non-homogeneity	Type B	0.00445	mm
u_{REST1}	Pinion Position "P"	Type A	0.00240	mm
u_{REST2}	SFT Backlash uncertainty	Type B	0.01520	mm
u_{LASH}	Combined uncertainty		0.01854	mm

Table 17

Standard uncertainty summary for 3.23 ratio torque audit measurement

Symbol	*u* Source	*u* Type	*u*	Units
u_{EVO}	Repeatability on workpiece	Type A	0.0151	Nm
u_{GV}	Process location	Type A	0.0361	Nm

Symbol	u Source	u Type	u	Units
u_{EVR}	Other torque correlation PTR	Type A	0.0266	Nm
u_{TTRA}	Combined uncertainty at Pinion		0.0473	Nm
u_{DTRA}	Combined uncertainty DTR for 3.23 Ratio $u_{TTRA} * Ratio$		0.1529	Nm

Uncertainty model summary.

Propagating the standard uncertainty predictions in the model provides prediction of process capability on Backlash and Differential torque to rotate (DTR). The uncertainty equation for the Pinion Side Shim Call, $u_{PS}^2 = u_{CAR1}^2 + u_{CAR2}^2 + u_{OAH}^2 + u_{BF}^2 + u_{\delta J}^2 + u_{Step}^2$ is applied to determine the resultant standard uncertaintyu_{PS}. The uncertainty equation for the Gear Side Shim Call, $u_{GS}^2 = u_{COV}^2 + u_{PSMeas}^2 + u_{Step}^2$ is used to determine resultant standard uncertainty u_{GS}. The equations below summarize the uncertainty contributions from the shim selection process that affect backlash.

$$u_{PS} = \sqrt{u_{CAR1}^2 + u_{CAR2}^2 + u_{OAH}^2 + u_{BF}^2 + u_{\delta J}^2 + u_{Step}^2}.$$

$$u_{PS} = \sqrt{.00510_{CAR1}^2 + .00438_{CAR2}^2 + .00783_{OAH}^2 + .00328_{BF}^2 + .00801_{\delta J}^2 + 0.00852_{Step}^2}$$

$$u_{PS} = 0.01594 \text{ mm}$$

$$u_{GS-LASH} = \sqrt{u_{COV1}^2 + u_{PSMeas}^2 + u_{Step}^2}$$

$$u_{GS-LASH} = \sqrt{0.00439_{COV1}^2 + 0.00367_{PSMeas}^2 + 0.00852_{Step}^2}$$

$$u_{GS-LASH} = 0.01026 \text{ mm}$$

The combined uncertainty for the backlash process is,

$$u_{Backlash-Process} = \sqrt{(-0.489)^2 u_{PS}^2 + (0.474)^2 u_{GS}^2} = 0.00919 \text{ mm}$$

The uncertainty equation for the shim error related to DTR, $u_{GS-DTR}^2 = u_{CAR1}^2 + u_{COV}^2 + u_{OAH}^2 + u_{PSMeas}^2 + u_{Step}^2$ is used to determine shim u_{GS-DTR}. The equations below summarize the uncertainty contributions from the shim selection process that affect DTR.

$$u_{GS-DTR} = \sqrt{u_{CAR1}^2 + u_{COV1}^2 + u_{PSMeas}^2 + u_{OAH}^2 + u_{Step}^2}$$

$$u_{GS-DTR} = \sqrt{0.00510_{CAR1}^2 + 0.00438_{COV1}^2 + 0.00367_{PSMeas}^2 + 0.00783_{OAH}^2 + 0.00852_{Step}^2}$$

$$u_{GS-DTR} = 0.01388 \text{ mm}$$

The combined uncertainty for the DTR process is,

$$u_{DTR} = \sqrt{(11.83)^2 u_{GS}^2} = 0.1642 \text{ Nm}$$

The measured process capability combines the process and measurement uncertainty. The audit standard uncertainty for DTR varies by ratio, $u_{DTRA-3.23} = 0.1684$Nm, $u_{DTRA-3.73} = 0.1911$Nm. The backlash audit standard uncertainty is $u_{LASH} = 0.01854$mm. The process observed variation is the combination of process variation and measurement variation, Table 18 summarizes the uncertainties associated with each process and audit.

Table 18

Combined standard uncertainty process capability

Symbol	*u* Source	Backlash	DTR 3.23	Units
$u_{Process}$	Process standard uncertainty	0.00919	0.1642	mm/Nm
u_{Audit}	Audit standard uncertainty	0.01854	0.1529	mm/Nm
$u_{Observed}$	Observed standard uncertainty $\sqrt{u_{Process}^2 + u_{Audit}^2}$	0.02069	0.2244	mm/Nm

Symbol	u Source	Backlash	DTR 3.23	Units
	Acceptance Limits $\lvert U-L\rvert$	0.100	1.16	mm/Nm
C_p	Predicted process capability $\lvert U-L\rvert/(6 * u_{Observed})$	0.806	0.862	
	Predicted % Rework $z=\pm\left(\frac{\lvert U-L\rvert}{2}/u_{Observed}\right)$	1.6%	1.0%	

Research Question 2

Research Question 2 asks, can a measurement system uncertainty model be used to predict the backlash and torque-to-rotate capability of a shim-selection measurement system? The method to answer this question is a comparison of product data to a MCM simulation applying the uncertainties developed answering Research Question 1.

Statistical Power and Effect Size

Research Question 2 compares model results with production data collected from the Supervisory Control and Data Acquisition (SCADA) system. This system permits the acquisition of a significant amount of data. To perform the comparison analysis it is desirable to collect data that is not influenced by variables not included in the study, ideally data collected for a continuous production run isolated to one part type. Given the study criteria of a Type I error of $p<.05$ an analysis of the amount data required to detect an effect for both variables of interest is required.

Effect size in literature is often classified as small, medium, and large (Cohen, 1992). Warner (2013) recommends distinguishing between statistical significance and practical significance for a study effect. It was possible to estimate a practical effect that is meaningful to the shim selection process. The shim selection process often operates at less than 100% first time acceptance (FTA), a value of 97.5% is common, resulting in 2.5% of the axles requiring replacement of the shims to achieve audit acceptance. The production shim selection process of this study typically operates near this FTA level. To establish an effect size for this system, a shift of ±1.5% in first time acceptance would be a meaningful change, for example the difference between 96% and 97.5%, or between 97.5% and 99% first time acceptance. Applying this as criteria, a standard deviation is calculated for the acceptance upper and lower acceptance limits assuming a centered two-tailed distribution. For 99% first time acceptance $z(.995) = 2.58$, for 97.5% first time acceptance $z(.9875) = 2.241$. The process standard deviation corresponding to 99% acceptance is calculated by dividing the half of the tolerance limit by the z value. The result is this ratio of standard deviation is the ratio of z values, the square of this ratio is the variance ratio that may be used to calculate a meaningful effect size for the F-Test of variances. To move from 97.5% FTA to 99% FTA a variance ratio of 0.76 is required, to move from 96% FTA to 97.5% FTA a variance ratio of 0.84 is required. The effect size for the F-Test is considered as the greater value of 0.84.

A similar approach is used to determine the Cohen d value for the mean comparison. A mean shift that corresponds to a 1.5% change in FTA, a single tail change from $z(.9875) = 2.241$ to 97.25% calculated, $z(.9725) = 1.92$. This change in z value multiplied by the standard deviation provides a measure of the mean shift corresponding to this change, 0.007 mm in backlash and 0.08 Nm in DTR can be used to calculate a Cohen d for each variable. The Cohen d for Backlash and DTR is 0.31. These values are used for further analysis of effect and power.

Production Data Screening

To validate the data a screening process was used to identify outliers and assess the normality of the data. The data was acquired by collecting results for 2,000 consecutive parts during normal production. The data included three separate axle models differing by gear ratio. A sort was conducted to include only parts first time through the process and to exclude parts with incomplete data sets. This resulted in 1,721 cases, 964 cases of 3.23 ratio models and 757 cases of 3.73 ratio models. The 3.23 ratio model data set was selected for analysis as providing the largest number of cases.

Initial data screening of the independent measurement variables identified a bimodal distribution in the independent variable CAR1 shown in Figure 18. The Carrier bimodal distribution is common to all models, attributed to two separate machine sources that produce the Carrier. For the purpose of data validation, the CAR1 is divided into two groups, Group A below 102.240 mm, and Group B above 102.240 mm.

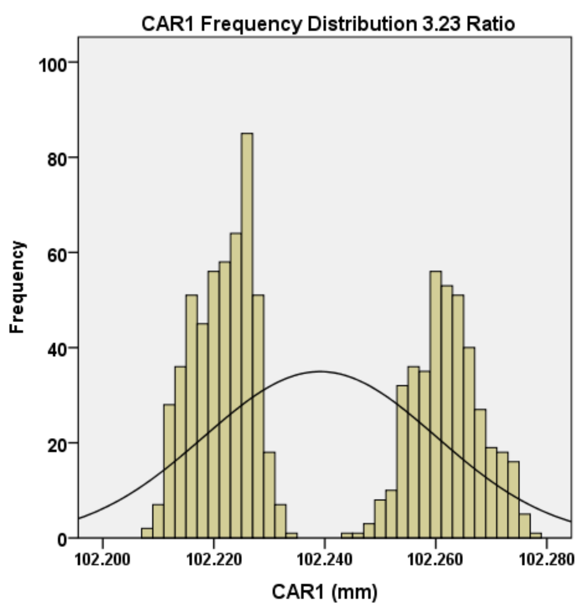

Figure 18. Production data bimodal distribution of CAR1 measurements

The independent variable distributions all appeared normally distributed, excess kurtosis for variables was less than 1.01 units. To eliminate data outliers the approach of 1.5 times the difference between the 25th and 75th percentile, defined as H-Spread, is added to the 75th percentile and subtracted from the 25th percentile (Warner, 2013, p. 156). This process eliminated 41 cases as independent variable outliers. The remaining 923 cases included two dependent variable outliers. Case 619 is an outlier in both DTRERR with a 3.4 H-Spread ratio and AVGLASH with a 2.7 H-Spread ratio. The second case excluded as a dependent variable outlier was case 593 with an AVGLASH H-Spread ratio of 3.0. The final data set included 921 cases.

The statistical power based on the 921 data sample size is calculated using G*Power Version 3.1 (Faul, Erdfelder, Lang, & Buchner, 2007). The t-test power analysis uses 921 samples for the data and an assumed 3,000 samples from the MCM and the Cohen d effect size of 0.31 previously determined for comparison of sample means. The calculated power is >.999, the risk of Type II error is negligible. For the variance F-Test, power is calculated again assuming 921 samples and 3,000 samples from the MCM, the variance ratio effect of 0.84 previously calculated is used for the power calculation. The calculated power of .91 indicates an acceptable 9% risk of Type II error. This verified that the sample set of 921 cases provided adequate power for this analysis.

The independent variable data statistics with the outliers eliminated is shown in Table 19. The CAR2 and CAR1 Group A variables exceed the 2.0 *z* ratio of skewness divided by Standard Error of skewness, z_{CAR2}= 2.4 and $z_{CAR1\text{-}GRPA}$= 2.2. This slight skew does not affect the results as each case is separately evaluated. All of the independent variable distributions are platykurtic, with CAR1 is the most significant with excess kurtosis of -0.844 and standard error of kurtosis

0.216. This is acceptable and is attributed to the small spread in the variable, 25 microns total, relative to the resolution of the data.

Table 19

Production data Independent Variable statistical summary

		Mean	Std. Dev.	Skewness		Kurtosis	
Variable	N	Statistic	Statistic	Statistic	Std. Err.	Statistic	Std. Err.
CAR1GRPA	509	102.221 mm	.00532	-.239	.108	-.844	.216
CAR1 GRP B	412	102.262 mm	.00618	.065	.120	-.327	.240
CAR2	921	42.019 mm	.00735	-.196	.081	-.190	.161
COV	921	55.532 mm	.01791	.054	.081	-.327	.161
DELTAJ	921	0.1508 mm	.02671	-.008	.081	-.262	.161
OAH	921	151.644 mm	.02551	-.073	.081	-.120	.161
BF	921	48.991 mm	.02904	.104	.081	-.250	.161

The statistics for the dependent variables of AVGLASH and the error in setting differential torque DTRERR are summarized in Table 20. Both dependent variables are normally distributed with z ratio of kurtosis and skewness less than 2.0. Backlash mean of 0.187 mm is 0.007 mm above the target, while the mode is at the target of 0.18 mm. The data mean shift or bias in the resultant backlash average in the data is at the effect size threshold of 0.007 mm established as part of this study. This indicates that the production process is not centered at 0.18 mm. The variation of differential torque to rotate from the target is 0.007 Nm, less than 0.6% of the total range and well below the practical effect size of 0.083 Nm established. The data set of 921 cases was the baseline for comparison with the Monte Carlo simulation for the two dependent variables.

Table 20

Production data Dependent Variable statistical summary

		Mean	Std. Dev.	Skewness		Kurtosis	
Variable	N	Statistic	Statistic	Statistic	Std. Err.	Statistic	Std. Err.
AVGLASH (mm)	921	.187	.02191	-.114	.081	-.218	.161
DTRERR (Nm)	921	.007	.21926	-.048	.081	-.137	.161

Monte Carlo simulation.

The simulation process follows the production gauging process for each case or part in the simulation. The simulation starts with a true value for each independent variable. An error based on the combined standard uncertainty is added to the true value to create an observed value. The simulation Excel spreadsheet follows the assembly process propagating error that results in a value for the two dependent variables of Differential Torque to Rotate Error and Backlash. A sample calculation from the simulation is included in Appendix D.

For each independent measurement, variable true values are simulated based on a normal distribution. To generate the true values a process Capability Index defined by; $C_p = |U - L|/(6 * \sigma)$ where σ is the process standard deviation (AIAG, 2010) was used. The target process control at this manufacturer is$C_p = 2.0$, this value is applied to the upper and lower limits for each variable true value. The analysis is based on the uncertainty around the measurements so that the process and analysis is robust to the MCS true values. Table 21 summarizes the upper (U) and lower (L) product limits averaged to determine the mean and the standard deviation for each variable true value determined by $\sigma = |U - L|/(6 * C_p)$. Independent Variable OAH and BF true values were calculated by adding the tolerance of each individual component. Variable DELTAJ has no specific tolerance, a mean of 0.15 mm and range $|U - L|$ of 0.30 mm were included for calculation of DELTAJ true values in the simulation. The individual case simulated true values are developed using the NORMINV function in Excel. The distribution is based on the mean and standard deviation of Table 21, $NORMINV(RAND(\quad), \mu, \sigma)$. The RAND Excel function generates a random true number between 0 and 1, providing a probability for each case calculation. This method is used in the simulation to generate the normal distribution true values for each independent variable.

Table 21

Independent variable true value MCM simulation characteristics

Variable	U (mm)	L (mm)	Mean μ (mm)	Std. Dev. σ (mm)
CAR1	102.300	102.100	102.200	0.016667
CAR2	42.025	41.975	42.000	0.004167
COV	55.580	55.480	55.530	0.008333
DELTAJ	0.300	0.000	0.150	0.025000
OAH	152.000	151.400	151.700	0.050000
BF	49.335	48.935	49.135	0.033333

To simulate the independent variable observed measured values the measurement uncertainty error is added to a simulated true value (Coleman & Steele, 2009). The uncertainty error is based on the standard uncertainty for each independent variable developed in answering Research Question 1. The individual simulation case measurement errors are developed using the NORMINV function in Excel, applying a mean of 0.0 and standard deviation value that corresponds to the standard uncertainty, and the RAND function to generate the normal distribution, $NORMINV(RAND(\quad), 0.0, u_i)$. The uncertainty for each independent variable is listed in Table 22.

Table 22

Independent variable MCM simulation measurement standard uncertainty

Variable	Reference RQ1	standard uncertainty *u*	Units
CAR1	Table 5	0.00510	mm
CAR2	Table 6	0.00438	mm
COV	Table 7	0.00439	mm
OAH	Table 9	0.00783	mm
BF	Table 10	0.00328	mm
DELTAJ	Table 11	0.00800	mm
Shim Measure	Table 8	0.00367	mm

The simulation uses observed values to select a shim from the incremental shims available and calculates an installed shim for each case. This installed shim is then compared with the True Value ideal shim gap. The difference between the true value shim gap and the installed shim constitutes the shim call process error. Applying the coefficients developed in answering Research Question 1, the effects on Differential Torque to Rotate and Backlash attributed to the shim error are calculated for each case in the simulation as, $DTRERR_{SHIM} = 11.83 * (GS_{ERROR})$, and $LASHERR_{SHIM} = (-0.489) * GS_{ERROR} + 0.474 * PS_{ERROR}$.

The process uncertainty simulated for each case includes in-process torque measurements that contribute to error in reported torque results. Two in-process measurements contribute to measurement uncertainty; Pinion Torque to Rotate (PTR) and Differential Torque to Rotate Process (DTRP). To represent the distribution for PTR and DTRP true values, a normal distribution is applied to the simulation. The mean selected for the simulated true value was the mean of the larger data set of 1953 cases. The standard deviation is based on the bearing manufacturers projected Upper and Lower limits as a six standard deviation range $\sigma = |U - L|/(6)$. The individual simulation cases are populated using the $NORMINV(RAND(\quad), \mu, \sigma)$ function in Excel in the same manner described for the independent variables. For PTR a mean true value of 1.7 Nm and $\sigma = 0.14$, and DTRP 2.6 Nm and $\sigma = 0.14$ is used in the simulation. Table 23 summarizes the in-process variable limits used in the MCM simulation.

Table 23

In-process variable true value MCM simulation characteristics

Variable	U (Nm)	L (Nm)	Mean (Nm)	Std. Dev.
PTR	2.125	1.275	1.7	0.14
DTRP	3.025	2.175	2.6	0.14

To represent the distribution for PTR and DTRP uncertainty values and error for each case the normal distribution is applied to the simulation. The individual simulation case PTR and DTRP measurement errors are developed using the NORMINV function in Excel. Applying a mean of 0.0 and standard deviation value that corresponds to the standard uncertainty, and the RAND Excel function to generate the normal distribution, $NORMINV(RAND(\quad), 0.0, u_i)$. From Table 15 above, the uncertainty for the in-process measurement variable PTR is 0.0320 Nm, and for DTRP is 0.0542 Nm.

The process uncertainty simulated for each case also includes uncertainty resulting from the gear manufacturing process. The process targets a mean backlash and the process variation is about that mean value. As part of answering research Question 1, a Type B uncertainty u_{REST2} based on normally distributed process limits was included in determining the backlash audit uncertainty shown previously in Table 16. The MCM simulation provides a method to include this uncertainty based on a Type A analysis. To simulate that variance production data available from Single Flank Test (SFT) of the hypoid gears included in this study was applied. The SFT data includes a Total Run-out measurement that combines total accumulated pitch variation and pitch line run-out (Smith, 1985). For the simulation data, this was applied to simulate the error effects on backlash measurement production using SFT total run-out data for the study 3.23 ratio axles. SFT data for 1337 parts were analyzed using SPSS to assess normality of the distribution that is shown in Figure 19. Applying the normality assessment criteria of 2.0 *z* ratio of skewness 0.729 divided by Standard Error of skewness 0.067, $z_{\mathrm{SFT\ RUNOUT}} = 10.9$ does not pass the normality test. To provide a simulation input for this uncertainty SPSS Descriptive Statistics Q-Q Plots is applied to model Shape Factor α and Scale Factor θ for a Gamma Distribution fit to the SFT data. The SPSS output estimates Shape Factor of 8.860, a Scale Factor of 195.973, and Median of 0.043 mm for the Gamma distribution. To simulate this variable in the Monte Carlo simulation two Excel functions are combined. The GAMMAINV function is applied to simulate error values. A random probability number provides a probability input to the GAMMAINV function to generate a random error that follows a Gamma distribution, $GAMMAINV(RAND(\quad), \alpha, 1/\theta)$. The increase in Total Run-out value results in a decrease in measured backlash for the assembly (Smith, 1985). To include this error the inverse random value is distributed about the median value around the target backlash of 0.180 mm. The result is the True Value of the backlash prior to assembly.

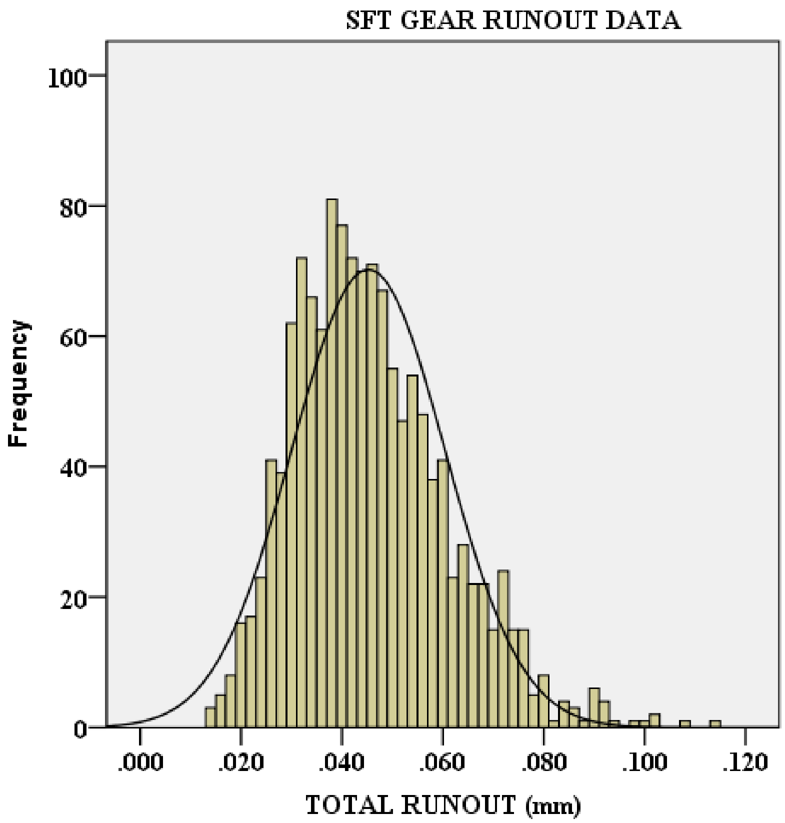

Figure 19. Single Flank gear run-out data histogram for 3.23 Ratio axle

The simulation proceeds to calculate a True Value for both dependent variables for each case. To calculate the True Value for the assembly DTR measured at the Pinion, the error resulting from shim call is added to the DTRP true value, creating a DTR True value in the assembly. This value is divided by the ratio and then added to the PTR true value to provide a true value for the measured total torque to rotate (TTR). To calculate the True Value for the assembly backlash the process error resulting from shim call is added to the SFT backlash true value.

The observed value for each dependent variable is calculated by adding the audit measurement error to the true value for each case based on the standard uncertainty. The standard for backlash excludes the Type B u_{REST2} associated with Single Flank Testing as described above. For TTR, the standard uncertainty u_{EVO} andu_{EVR} associated with TTR measurement from Table 17 above is included. The MCM observed DTR is determined by subtracting the observed process PTR from the observed audit TTR, multiplying time the ratio to determine the audit DTR. The difference between the audit DTR and MCM process DTRP generates an error for each case. The error is the result of the backlash variation about the target mean of 0.180 mm Normal Backlash.

Table 24

Audit measurement MCM simulation standard uncertainty

Symbol	u Source	Reference RQ 1	u	Units
u_{LASHA}	Backlash Audit uncertainty	Table 16*	0.0106	mm
u_{TTRA}	TTR Audit uncertainty	Table 17*	0.0306	Nm

*Note. Includes only standard uncertainties associated with the audit process

Monte Carlo Simulation Validation

Estimating the simulation model error is the objective of MCM validation (Coleman & Steele, 2009, p. 194). The primary MCM validation is conducted by making a comparison of the MCM results to data in answering Research Question 2. As additional validation several methods that are recommended in literature are used. First is confirmation that the numerical code is accurate in calculating the results as part of MCM validation (Coleman and Steele, 2009). Second is the determination of the number of Monte Carlo trials necessary to achieve the required model accuracy (JCGM, 2008c). Third, Coleman and Steele (2009) recommend determining error through multiple model iterations with varying input variables in repeated runs as a validation method. Finally, the GUM (JCGM, 2008c) recommends validating the MCM simulation by comparing results between the GUM uncertainty framework and Monte Carlo Method. This study included all of the validation methods described.

Numerical evaluation recommended by Coleman and Steele (2009) is relative to the convergence of complex iterative numerical calculations. The MCM of this study is an additive model, not subject to numerical solution error. The numerical validation consists of verification that the model functions are used properly. To perform this validation, the MCM results and the numerical calculations were validated, the results are summarized in Appendix D.

To validate the number of MCM iterations, an adaptive approach was selected by assessing the stability of the standard deviation of the desired parameter for stabilization (Coleman & Steele, 2009). Coleman and Steele (2009) recommend evaluating the standard deviation of the MCM result for convergence within 1-5%. Given that this simulation is an adaptation of the uncertainty method applied to shim selection where the parameters of interest include standard deviation, the percentage of change in standard deviation is monitored for stabilization. The monitoring approach applies a 200 iteration running average of the change in standard deviation, $\%\,RUN\ AVERAGE = \left[\sum_{i=j}^{j+199}(\sigma_i - \sigma_{i-1})/\sigma_i\right]/200$. The results are shown in Figure 20 where after iteration 2180 the model percent running average is converged within 0.005%, and after iteration 4000, the model is converged within 0.0025%. Based on this study, the number of iterations selected is 4,000.

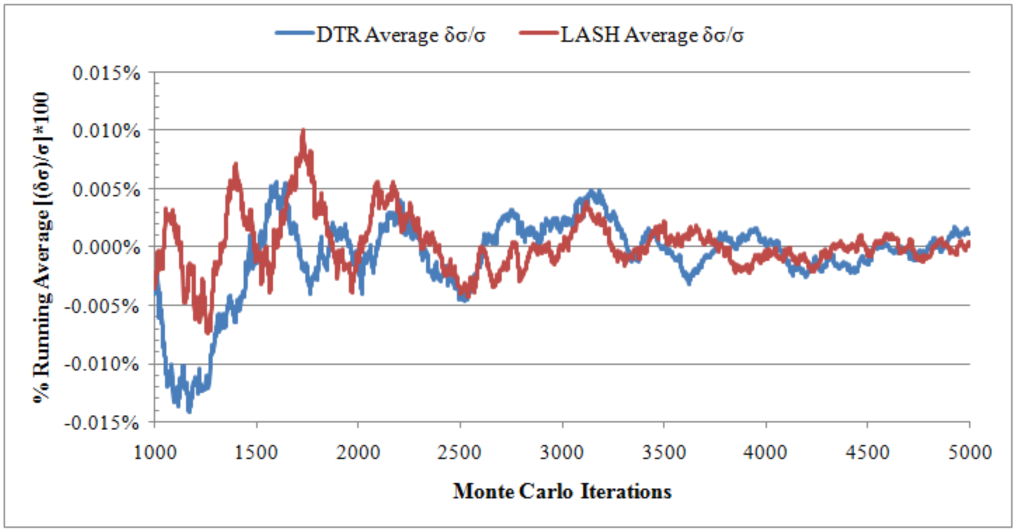

Figure 20. MCM simulation standard deviation stability based on number of iterations

A comparison of the Independent Variable observed values from the MCM to Production Data provides another validation of the MCM simulation. A statistical summary of the data results are shown in Table 25, and a comparable summary of the MCM observed values are shown in Table 26. The production data sets all are platykurtic because of the small sample size, where the MCM large sample size distributions follow a normal distribution are normal or leptokurtic. The difference in standard deviation and mean for CAR1 is a bias in the single machining center Carrier Group B data discussed earlier, both CAR1 distributions are normal. The COV production data includes a larger standard deviation than the MCM, attributed to multiple machining centers producing the part that are not centered about the same mean. The MCM Independent variables normal distribution provided a seed to the simulation that adequately represents to process to evaluate the effects of uncertainty.

Table 25

Production data Independent Variable observed value statistics

	*CAR1	CAR2	COV	DELTAJ	OAH	BF
N Valid	412	921	921	921	921	921
Mean (mm)	102.262	42.019	55.532	0.151	151.644	48.991
Std. Error of Mean	0.0003	0.0002	0.0006	0.0009	0.0008	0.0010
Median (mm)	102.262	42.020	55.531	0.151	151.644	48.991
Std. Deviation	0.0062	0.0074	0.0179	0.0267	0.0255	0.0290
Skewness	0.065	-0.196	0.054	-0.008	-0.073	0.104
Kurtosis	-0.327	-0.190	-0.327	-0.262	-0.120	-0.250
Minimum (mm)	102.244	42.000	55.480	0.074	151.573	48.912
Maximum (mm)	102.277	42.039	55.584	0.228	151.714	49.070

*CAR1 GROUP B

Table 26

MCM simulation Independent Variable observed value statistics

	SIMCAR1	SIMCAR2	SIMCOV	SIMδJ	SIMOAH	SIMBF
N Valid	5000	5000	5000	5000	5000	5000
Mean (mm)	102.200	42.000	55.530	0.150	151.699	49.135
Std. Error of Mean	0.0002	0.0001	0.0001	0.0004	0.0007	0.0005
Median (mm)	102.200	42.000	55.530	0.150	151.698	49.135
Std. Deviation	0.0174	0.0060	0.0094	0.0262	0.0511	0.0336
Skewness	0.044	-0.040	0.026	0.022	-0.072	-0.015
Kurtosis	0.003	-0.085	0.096	0.138	1.310	-0.007

	SIMCAR1	SIMCAR2	SIMCOV	SIMδJ	SIMOAH	SIMBF
Minimum (mm)	102.139	41.978	55.497	0.049	151.232	49.009
Maximum (mm)	102.274	42.021	55.565	0.286	151.875	49.260

A comparison of the MCM in-process PTR and DTRP observed values to Production Data validate the MCM simulation as shown in Table 27. The production data sets all are skewed positively where the MCM large sample size distributions follow a normal distribution. The MCM Independent variables normal distribution provides a representative distribution of the in-process variables to the simulation. The accuracy in predicting the in-process variables provides further validation of the MCM simulation.

Table 27

Comparison of production data in-process variables to MCM observed values

		PTR	SIMPTR	DTRP	SIMDTRP
N	Valid	921	5000	921	5000
Mean (Nm)		1.70	1.70	2.59	2.60
Std. Error of Mean		0.0046	0.0020	0.0054	0.0021
Median (Nm)		1.69	1.70	2.58	2.60
Std. Deviation		0.1385	0.1432	0.1639	0.1503
Skewness		0.195	-0.012	0.180	0.026
Kurtosis		-0.364	0.093	0.277	-0.027
Minimum (Nm)		1.31	1.08	2.04	2.01
Maximum (Nm)		2.17	2.22	3.16	3.27

The stability of the MCM was validated using a modified approach of combined input variables as discussed by Coleman and Steele (2009). The MCM model is run 150 separate trials of 4,000 iterations for each trial. The results are summarized in Table 28. This approach simulates varying input measurements, or independent variables, to the model. Assessing the error of the predicted means and standard deviation of backlash and DTR was used to develop a confidence interval for each run of the model. As part of the model validation, the MCM results for calculated standard deviation are compared to the GUM uncertainty framework using uncertainties previously listed in Table 18. The Gum uncertainty framework prediction for backlash was $u_{Lash\ Observed} = 0.02068$. Comparing this to the MCM 150 run value of SD = 0.02036 results in less than 0.5% change in reject rate. Similarly $u_{DTR\ Observed} = 0.2249$ compared to the 150 run simulation SD=.2236 is less than 0.05% change in-process reject rate.

Table 28

Average MCM results for 150 simulation runs of 4,000 iterations

Variable	Mean	Standard Error
Average of Backlash (mm)	0.1783	0.00002
Average SD Backlash (mm)	0.02036	0.00002
Average of DTR Error (Nm)	-0.0002	0.00029
Average SD DTR Error (Nm)	0.2236	0.00021

MCM Simulated Data for Research Question 2

Research Question 2 asks if a measurement system uncertainty model can be used to predict the backlash and torque-to-rotate capability of a shim-selection measurement system? The question is answered by comparing the Monte Carlo simulation to the production data. To answer research Question 2 a 4,000 case MCM was used to generate a comparison data set for the hypothesis testing. The comparison of MCM results and production data for the two dependent variables is shown in Table 29. The simulation data tends to be more leptokurtic because of the larger sample size, kurtosis for average lash in the data is -.218 while the simulation is .163. Both distributions are platykurtic for the differential torque error, with the production data more platykurtic with kurtosis of -.137 compared to the simulation value of -.089. The skew of backlash distribution is negative for both the simulation and the production data. The source of the skew in the simulation is the gamma distribution for gear run-out contribution to uncertainty. A comparison of the MCM and production data distributions of backlash and DTRERR are shown in Figure 21. The backlash distributions appear similar, including the slight negative skew. A comparison the MCM and production data distributions of DTRERR, both distributions appear normally distributed and centered about the mean of 0.0 Nm.

Table 29

Dependent variables for Research Question 2

	Data DTRERR (Nm)	MCM DTRERR (Nm)	Data AVGLASH (mm)	MCM AVGLASH (mm)
N Valid	921	4000	921	4000
Mean	.0070	-.0005	.187	.178
Median	.0172	-.0004	.19	.18
Std. Deviation	.21926	.22508	.02191	.02039
Skewness	-.048	-.047	-.114	-.289
Kurtosis	-.137	-.017	-.218	.220
Minimum	-.573	-.992	.12	.08
Maximum	.787	.831	.24	.24

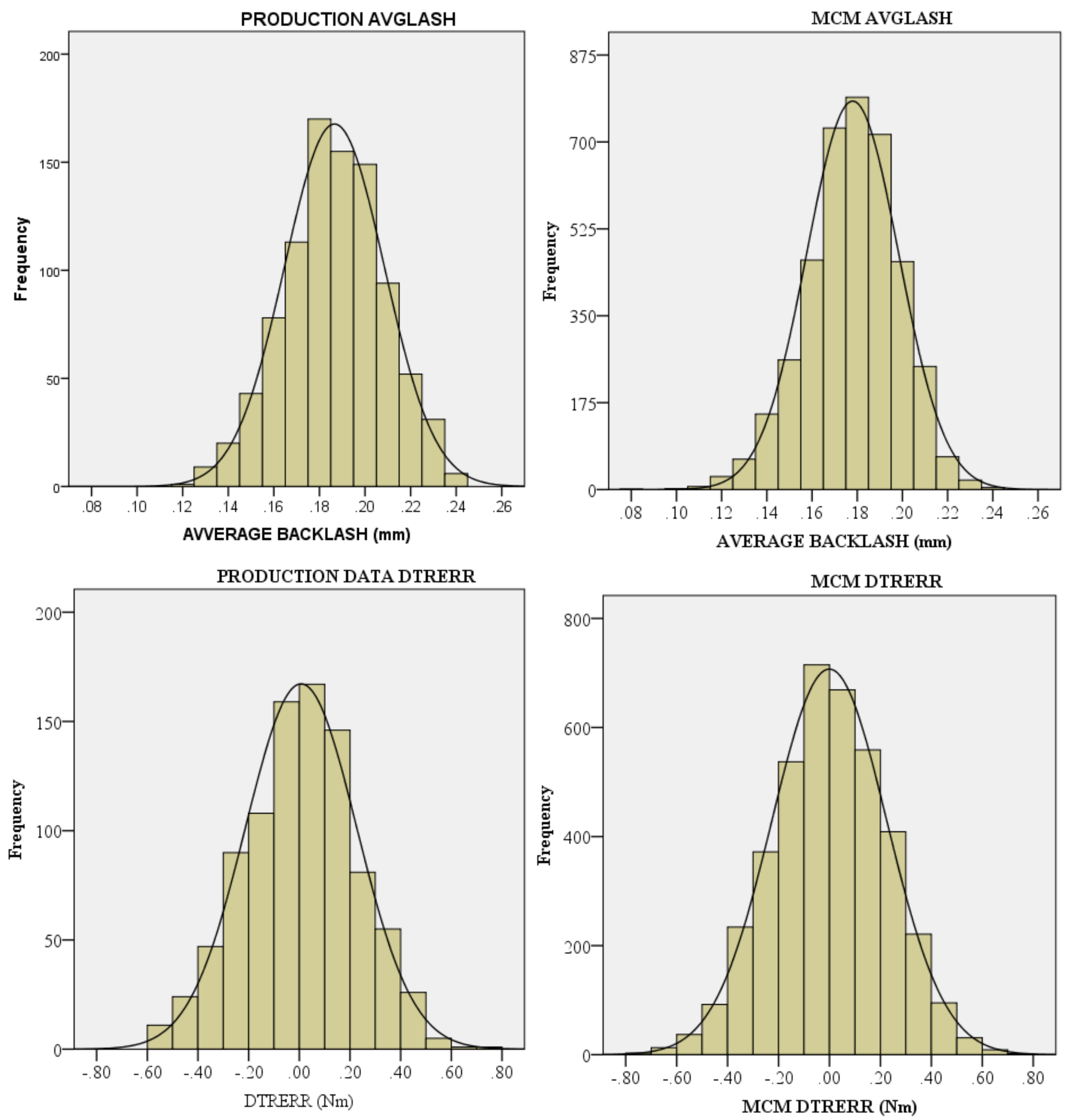

Figure 21. Dependent Variable production data and MCM Simulation histograms

Hypotheses Testing

The first hypothesis compares means of the two samples using t-Test for two sample means.

H_{O1}: There is no significant difference between the Means of the uncertainty prediction model and actual test data in Backlash Audit.

H_{A1}: There is significant difference between the Means of the uncertainty prediction model and actual test data in Backlash Audit.

To compare the means of the simulation to the production data a two-sample t-Test assuming unequal variance compares the mean values of the two tests. The result, $t(1312) = -10.9$ ($p < .001$) indicates a statistical difference in the means, rejecting the null hypotheses that the means are equal. The statistical mean difference is analyzed using the standardized Cohen effect size d=.39 which is between a small and medium effect (Cohen, 1992). The actual Mean difference of 0.009 mm is above the practical effect size established for this study. The MCM model is within .001 mm of the 0.18 mm target, the rejection is a result of the production process not being centered.

The second hypothesis compares the variance of backlash for the two samples using the F-Test for two sample variances.

H_{O2}: There is no significant difference between the Variance of the uncertainty prediction model and actual test data in Backlash Audit.

H_{A2}: There is significant difference between the Variance of the uncertainty prediction model and actual test data in Backlash Audit.

The F statistic for the samples, $F(3999,920) = 0.87$ ($p = .002$), less than F-Critical for the sample sizes, F-Critical $(3999,920) = 0.91$. This indicates a statistical difference, resulting in a rejection of the null hypotheses that the variances are equal. The large amount of data from the MCM provides enough statistical power to detect small effects. The practical significance is determined by the effect size. This difference is not less than the practical variance ratio of 0.84 previously established. The standard deviation of the data would result in a predicted reject rate of 2.2% for the production data and for the MCM 1.4%, this 0.8% difference is not practically significant.

The third hypotheses compares the means of the two samples for error in measuring differential torque to rotate using the t-test for two sample means.

H_{O3}: There is no significant difference between the Means of the uncertainty prediction model and actual test data in Audit Total Torque to Rotate.

H_{A3}: There is significant difference between the Means of the uncertainty prediction model and actual test data in Total Torque to Rotate.

To compare the means of the simulation to the production data a two-sample t-Test assuming unequal variance compares the mean values of the two tests. The result $t\ (1402) = -0.93$ ($p = .35$) indicates no statistical difference in the means and a failure to reject the null hypotheses that the means are equal. The mean error of the simulation and the sample data are both centered about zero. This indicates the process is targeted properly for the production data, and that the MCM is producing the correct result.

The hypothesis compares the variance of differential torque error for the two samples using the F-Test for two sample variances.

H_{O4}: There is no significant difference between the Variance of the uncertainty prediction model and actual test data in Total Torque to Rotate.

H_{A4}: There is significant difference between the Variance of the uncertainty prediction model and actual test data in Total Torque to Rotate.

The F statistic for the samples, $F(920,3999) = 0.95$ ($p = .16$), greater than F-Critical for the sample sizes, F-Critical $(920,3999) =0.902$. This indicates there is no statistical difference, resulting in a failure to reject the null hypotheses that the variances are equal. The practical significance determined by the effect size is not less than the practical variance ratio of 0.84 previously established. The standard deviation of the data would result in a predicted reject rate of 0.8% and for the MCM 1.0%. This difference is not practically significant, the model predicted the performance of the system first time acceptance with less than 0.5% difference.

Research Question 3

Research Question 3 asks can a measurement system uncertainty model be used to determine the acceptance limits for an individual in-process shim-selection measurement apparatus? The question is answered by comparing results of two iterations of the Monte Carlo simulation. The first iteration is run with a repeatability GR&R of 10% over the 0.100 mm tolerance range for the COV Independent variable, the second with GR&R of 50%.

The Monte Carlo simulation results are compared from two separate runs varying repeatability on a workpiece μ_{EVO}. The first run applies the 10% GR&R assumption in the equation for repeatability studies, $\%GR\&R = (5.15 * \mu_{EVO})/|U - L|$. Rearranging to determine the associated workpiece repeatability uncertainty for varying %GR&R, $\mu_{EVO} = |U - L| * \%GR\&R/(5.15)$. The specification limit for COV $|U - L|$ is 0.10 mm, the resultant μ_{EVO} for 10% GR&R is 0.001942, for 50% GR&R 0.009709. Substituting these values into the overall uncertainty for COV in Table 7 above the μ_{COV} for 10% GR&R is 0.00477 and for 50% GR&R is 0.01064. By comparison, the 10% GR&R run for backlash resulted in a Mean=0.178, SD=0.02018, and the 50% GR&R run Mean=0.178, SD=0.02124. The 10% run for DTR error resulted in a Mean=0.004, SD=0.2245, the 50% GR&R run Mean=-.001, SD=0.2545. This data was used to analysis the following hypotheses.

H_{O5}: There is no significant difference between the Means of the uncertainty prediction model Audit Backlash with COV measurement capability at 10% and 50%.

H_{A5}: There is significant difference between the Means of the uncertainty prediction model Audit Backlash with COV measurement capability at 10% and 50%.

The means of the two simulation runs are analyzed using a t-Test to compare the mean values of the two MCM simulation runs assuming unequal variance. The result $t(7977) = 0.09$ ($p = .93$) indicated there is no statistical difference in the mean in backlash which results in a failure to reject the null hypothesis that the means are equal. This is as expected, uncertainty is centered about zero for both runs, and the variation on the average result is expected to be insignificant.

To analyze hypotheses 6 the variance of backlash for the two samples used the F-Test for two sample variances.

H_{O6}: There is no significant difference between the Variance of the uncertainty prediction model and Audit Backlash with COV measurement capability at 10% and 50%.

H_{A6}: There is significant difference between the Variance of the uncertainty prediction model Audit Backlash with COV measurement capability at 10% and 50%.

The F statistic for the two simulation runs, $F(3999,3999) = 0.903$ ($p = .001$), indicating that there is a statistical difference and rejecting the null hypotheses that the variances are equal. The practical significance is determined by the effect size, this difference is not less than the practical variance ratio of 0.84 previously established. The standard deviation of the data would result in a predicted reject rate of 1.3% for the 10% GR&R data and 1.9% for the 50% GR&R data, this difference is not practically significant. A degradation in performance of that gauge would not a result in a discernable difference in backlash process performance.

The two separate simulation runs were evaluated to compare means of DTR error to test hypotheses 7.

H_{O7}: There is no significant difference between the Means of the uncertainty prediction model Audit Torque-to-Rotate with COV measurement capability at 10% and 50%.

H_{A7}: There is significant difference between the Means of the uncertainty prediction model Audit Torque-to-Rotate with COV measurement capability at 10% and 50%.

The means of the two simulation runs are analyzed using a t-Test to compare the mean values of the two simulation runs assuming unequal variance. The result, $t(7876) = 1.01$ ($p=.32$) indicated there is no statistical difference in the mean which results in a failure to reject the null hypotheses that the means are equal. This result is as expected, a difference in GR&R performance should not result in a process mean shift.

To analyze hypotheses 8 the variance of DTR error for the two samples used the F-Test for two sample variances.

H_{O8}: There is no significant difference between the Variance of the uncertainty prediction model Audit Torque-to-Rotate with COV measurement capability at 10% and 50%.

H_{A8}: There is significant difference between the Variance of the uncertainty prediction model and Audit Torque-to-Rotate with COV measurement capability at 10% and 50%.

The F statistic for the two simulation runs, $F(3999, 3999) = 0.78$ ($p < .001$), indicated that there is statistical evidence to reject the null hypotheses that the variances are equal. The variance is significantly affected by the GR&R of the cover dimension measurement. The practical significance is determined by the effect size, this difference is less than the practical variance ratio of 0.84 previously established. The standard deviation of the data would result in a predicted reject rate of 1.2% for the 10% GR&R simulation, and 2.3% for the 50% GR&R

MCM simulation. This result is predictable when reviewing the uncertainty framework for DTR error. The uncertainty associated with DTR error includes fewer variables, the individual component of Cover measurement uncertainty is a significant portion of the process uncertainty. This is magnified when considering the sensitivity of DTR to shim selection error. Degradation in performance of the Cover gauge would result in a discernable practical difference in-process performance.

CHAPTER 5

SUMMARY, FINDINGS AND DISCUSSION

In automotive axle assembly, select fit shims or spacers are a common practice to control gear position and set bearing preload. The assembly equipment for this style axle typically integrates a complex measurement system into the assembly process to select the required shims. Such a measurement system combines component and subassembly measurements with post assembly audit measurements that audit gear position by measuring gear backlash, and bearing preload by measuring torque to rotate. To determine the shim-selection process suitability, a predictive measurement system analysis (MSA) method that correlates independent measurement variables to dependent output variables is required. In manufacturing, measurement systems are commonly assessed by GR&R studies that analyze measurement capability to process or drawing limits of an upstream manufacturing process. In automotive axle shim selection processes the measurement system is an integral part of the manufacturing process, therefore, there is a need to assess the measurement system based on the overall process capability. The lack of an identified performance prediction method to correlate input and audit measurements is the problem this research sought to address.

A review of the literature on the topic of measurement uncertainty confirmed that the techniques are commonly applied. A typically application is the expression of confidence intervals for reported measurements. Uncertainty methods used include conventional repeatability studies and other statistical analysis based on a series of observations, classified as Type A. A second classification not based on statistical analysis, classified as Type B, provides a method to include factors that contribute to measurement error but are not readily analyzed by statistical methods. Published literature on uncertainty analysis provides a framework for defining and propagating uncertainties, including the application of Monte Carlo Methods (MCM). This established uncertainty framework, though typically applied to determining measurement confidence intervals, appeared applicable as a predictive tool in automotive axle shim selection. This study sought to determine if uncertainty analysis methods are suitable as a predictive tool to correlate input and output measurements in automotive axle shim selection.

This study applied measurement uncertainty methods in a correlation study of component input measurements as independent variables, and resulting audit measurements as dependent variables in the axle shim-selection process. The uncertainty framework provided a technique to include factors beyond traditional repeatability studies that affect the process, including those not directly part of the measurement system. The purpose of this research was to apply and assess this method as a capability prediction tool of an axle shim-selection measurement system. The study included three sequential steps. First, the study developed an uncertainty model to characterize the relationship between independent variables and the dependent variables of backlash and bearing preload as measured by torque to rotate. Second, a Monte Carlo simulation compared the model prediction to a production measurement system. Third, the study applied the approach by assessing the effects of changing measurement capability for a specific independent variable on the output of the audit dependent variables. The study sought to answer the primary research question; can the application of uncertainty principles in measurement provide a method to predict performance of the shim-selection process in controlling backlash and torque to rotate for an axle shim-selection measurement process?

Findings

Research Question 1 asks how can uncertainty methods be applied to the axle shim selection process. To answer this question, an uncertainty model for a specific axle shim selection process was developed applying standard uncertainty categories described in *Statistical methods in process management -- Capability and performance -- Part 7: Capability of measurement processes,* Standard No. 22514 (ISO, 2012). The measurement uncertainty included Type A and Type B methods using propagation techniques published by; ISO Standard No. 22514, the *Evaluation of measurement data — Guide to the expression of uncertainty in measurement* No. 100:2008 (JCGM, 2008a), and Dietrich and Schulze *Measurement process qualification* (2011). The results of this study indicate this approach can be applied as a framework to identify the elements of uncertainty and sources of error for each variable in the process. Uncertainty methods permit consideration of multiple factors that are not included in a GR&R repeatability study. For several variables, uncertainty not normally associated with the measurement was a significant contributor to the process uncertainty. For example, the study indicated that uncertainty associated with controlling backlash in the shim selection process includes a significant contribution of error associated with gear lapping and Single Flank Testing.

Published uncertainty methods were used to propagate independent measurement uncertainty through the shim call equation to correlate error associated with the selected shim on the dependent variables. The uncertainty associated with the Pinion Side Shim was $u_{PS} = 0.01594$ mm and includes five of the independent measurement variables. The uncertainty associated with the 0.0254 mm incremental shim steps is the largest single contributor.

Similarly, the Gear Side Shim uncertainty that affects backlash was $u_{GS-LASH} = 0.01026$mm. Again, the largest single contribution is from the incremental shim step. The uncertainties in the shim call process that affect DTR are calculated through the Gear Side Shim call with a propagation equation that includes the elements that affect the bearing preload only. The uncertainty contribution of the shim selection process $u_{GS-DTR} = 0.01387$mm, again the largest single contributor being the uncertainty associated with the incremental shim step.

Linear regression analysis confirmed the relationship between DTR and total shim is linear, the regression model explaining greater than 90% of the variance. Using multiple regression analysis of the data confirmed the relationship between Backlash and shim changes is linear. The regression model explains greater than 95% of the variance and both the Gear-side coefficient and the Pinion-side coefficient were statistically significant.

The calculated uncertainty in backlash associated with shim selection process was $u = 0.00904$ mm. Comparing the backlash SD of the production process data sample 0.0219 mm, it is evident that more than 50% of the process variability is not part of the process for selecting the shim. Similarly, the DTR error from the process of determining the shim selection results in $u = 0.164$ Nm for Differential Torque while the production sample DTR error SD is 0.219 Nm. In this case, the shim selection process contributes 75% of the process variability. Combining this process uncertainty with the audit measurement uncertainties for the 3.23 ratio resulted in an expected uncertainty for backlash audit of $u_{BACKLASH} = 0.02062$ mm, and DTR audit of $u_{DTR} = 0.2309$ Nm. The uncertainty model developed in Research Question 1 predicted a first time process audit acceptance for backlash of 98.5% and for DTR 98.8%.

Research Question 2 asks, can a measurement system uncertainty model be used to predict the backlash and torque-to-rotate capability of a shim-selection measurement system? To answer this question the measurement uncertainties developed in answering Research Question 1 were included in a Monte Carlo Method (MCM) data simulation tool. A comparison of MCM simulated data to production data provided a validation for the uncertainty model. The large amount of data from the MCM simulation resulted in an “over-powered” study that detected statistical significance in hypothesis testing that was not practically significant. The study established practical significance for each hypothesis that is used to identify meaningful significance.

The first hypothesis was used to assess the simulation capability in predicting the backlash average. The null hypotheses that there is no significant difference between the Means of the uncertainty prediction model and actual test data in Backlash Audit was rejected. The model mean of 0.178 mm with a sample standard deviation 0.0204 and the data Mean of 0.187 mm with a sample standard deviation of 0.0219 results in a mean difference of 0.009 mm, which was significant having a probability of Type I error less than 0.001. This difference is associated with the production data not being centered on the target of 0.18 mm backlash whereas the simulation mean is nearly at the target. The standardized effect size of this difference d=.39 is between a small and medium effect (Cohen, 1992). The practical effect size determined to detect a 1.5% shift in first time quality is equivalent at 0.007 mm, the data and simulation mean difference is practically significant. The non-central data bias is not simulated by the MCM model, and may be part of a bias in one of the measurements during the production run that could have been compensated by refining the production offset. The purpose of this hypothesis test was an assessment of the model at targeting to the desired mean, the conclusion is that the model is adequately simulating the process mean for backlash.

The second hypothesis was used to determine the simulation capability to predict the variance of backlash. The null hypotheses that there is no statistical difference between the variance of the uncertainty prediction model and actual test data in Backlash Audit was rejected. The difference of the model mean variance of 0.000416 and the data variance of 0.000480 is significant, with probability of a Type 1 error less than .002. This is an indication that the model does not predict the shim selection backlash process variance. For this study an F-Ratio less than 0.84 was determined to have a significant effect of ±1.5% in first time acceptance. There is statistical significance but the effect is not practically significant. Quantifying the effect as an impact on process capability, for the data C_p (= 0.761) and the MCM is C_p (= 0.817), corresponding to a predicted process reject rate of 2.2% for the data and 1.4% for the MCM. Therefore, the conclusion is that though the null hypothesis is rejected, the effect of the difference is not significant and the model is a valuable tool in estimating backlash variance.

The third hypothesis was used to determine the simulation capability to predict the resultant DTR average. The result failed to reject the null hypotheses that there is no significant difference between the Means of the uncertainty prediction model and actual test data in DTR Audit error. The model mean of -0.0005 Nm with sample standard deviation of 0.225 and the data Mean of 0.007 Nm with a sample standard deviation 0.219 have a difference of 0.0075 Nm, which is not statistically significant considering the .18 probability of Type I error. The standardized effect size of this difference d=.034 is insignificantly small (Cohen, 1992). For this dependent variable, the process is targeted at the desired mean of 0.0 Nm. The purpose of the hypotheses test was an assessment of the model at targeting to the desired mean, the conclusion is that the model is adequately simulating the process mean for DTR.

The fourth hypothesis was used to determine the simulation capability to predict the variance of DTR error. The null hypotheses that there is no statistical difference between the variance of the uncertainty prediction model and actual test data in DTR Audit was accepted. The difference between the model mean of variance of $\sigma^2 = 0.0507$ and the data variance of $\sigma^2 = 0.0481$ is not significant based on the Type I error probability of .16. This is an indication that statistically the model does predict the shim selection DTR process variance. To determine the practical significance, an F-Ratio less than 0.84 was determined to have a significant effect of ±1.5% in first time acceptance. Quantifying the effect as an impact on process capability, for the data C_p(= 0.88) and the MCM is C_p(= 0.86), corresponding to a predicted process reject rate of 0.8% for the data and 1.0% for the MCM. The conclusion confirms a failure to reject the null hypothesis. The effect of the difference is not significant and the model is a valuable tool in estimating DTR variance. The results for Research Question 2 are summarized in Table 30.

Table 30

Summary of Results – Research Question 2

	Data	MCM	Statistic	Probability
Backlash Mean (mm)	0.187	0.178	t(1312) = -10.9	p<.001
Backlash SD (mm)	0.0219	0.0204	F(3999,920) = 0.87	P<.002
DTR Error (Nm)	0.0070	-0.0005	t (1402) = -0.93	p=.35
DTR Error SD (Nm)	0.2193	0.2251	F(920,3999) = 0.95	p=.16

A desired outcome of this research is the development of a technology management tool to assess shim selection measurement systems. Hypotheses five through eight were aimed at this goal by applying the simulation as a tool to predict the impact of a variation in the repeatability and reproducibility %GR&R on the independent variable COV. The null hypotheses five and seven concerned the variation on the means of Backlash and DTR error, that there is no significant difference between the Means of the uncertainty prediction model for the dependent variables with variation in COV %GR&R. For both dependent variables there was no significant statistical difference in the mean, Backlash [t(7977) = 0.09 (p = .92), and DTR error t(7876) = 1.01 (p = .32)]. There was a failure to reject the null hypotheses. The difference in Backlash Audit was negligible with less than 0.001 mm in backlash and 0.001 Nm in DTR. This confirmed that an unbiased increase in uncertainty in measurement does not result in a mean shift in the dependent variables.

The sixth and eighth hypotheses were used to determine the simulation capability to predict the variance of Backlash and DTR error. Null hypothesis six stated there is no significant difference between the Variance of the uncertainty prediction model and Audit Backlash with COV measurement capability at 10% and 50% was not rejected. The model mean of variance at 10% COV GR&R, i.e. $\sigma^2 = 0.00040$ and the variance at 50% COV GR&R, i.e. $\sigma^2 = 0.00044$ was statistically significant, [F(3999, 3999) = 0.90 (p<.001)]. Though statistically significant, the practical effect was negligible resulting less than 1% change in first time acceptance. This is an example of applying the model to establish criteria for the measurement apparatus acceptance. If the criteria is controlling backlash in the process, the change introduced by this measurement apparatus error is not significant. The technology manager could apply this information to optimize the system based on capital and operating cost.

Null hypothesis eight states there is no significant difference between the Variance of the uncertainty prediction model and Audit DTR error with COV measurement capability at 10% and 50% was rejected, the alternative hypotheses is accepted. The model mean of variance at 10% COV GR&R, i.e. $\sigma^2 = 0.0504$ and the variance at 50% COV GR&R, i.e.$\sigma^2 = 0.0648$ is significant, [F(3999, 3999) = 0.78 (p<.001)]. The F-Ratio is less than 0.84 criteria established, indicating a practical significance. The predicted reject rate for 10% GR&R is 1.0%, and for

50% GR&R it is 2.3%. This provides another example of applying the model to establish criteria for the measurement apparatus acceptance. For criteria of controlling DTR in the process, the error introduced by this measurement is statistically significant and has some practical significance.

Table 31

Summary of Results – Research Question 3

	COV GR&R 10%	COV GR&R 50%	Statistic	Probability
Backlash Mean (mm)	0.178	0.178	t(7977) = 0.09	p=.92
Backlash SD (mm)	0.0202	0.0212	F(3999,3999) = 0.90	p<.001
DTR Error (Nm)	0.004	-0.001	t (7876) = 1.01	p=.32
DTR Error SD (Nm)	0.224	0.254	F(3999,3999) = 0.78	p<.001

Implications of This Study

This correlation study indicates that the input and output measures of the shim selection process are influenced by a number of factors that are not part of the measurement system direct measurement process. Based on this research, the indirect process measurements, torque as a measure of bearing preload and backlash as a measure of gear position, contribute significantly to the process uncertainty and therefore process capability. The dynamic variability of the test article torque is a significant contributor to overall process uncertainty in assessing the actual bearing axial preload. The contribution of the workpiece repeatability (u_{EVO}), typically the only measurement assessment, explains less than half of the process uncertainty. The indirect measure of backlash to assess gear position, while the gear manufacturing uses position, contributes to uncertainty and process variation. The implication of this study is that a method to quantify and improve the control of these factors will be required to improve process capability.

The answers to Research Question 3 provide an example where a significant difference in percent GR&R makes no statistical difference in the predicted process capability in controlling backlash. The implication is that an analysis beyond GR&R studies for equipment is necessary to understand the sources of process variability. The results of the study support that measurement uncertainty propagation is a method to evaluate and analyze these factors. Based on the hypothesis testing, improving the existing measurement system apparatus GR&R will produce minor process capability improvements. The implication is that axle designs and axle manufacturing processes will need to consider control and measure of variables currently not included to improve this process. The technology manager could apply this information to influence the product design, optimize the system, and make trade-offs on capital and operating cost.

This study applied uncertainty methods to correlate input and output measures for an automotive axle shim selection process. Implicitly, the technique of allocating standard uncertainty to a select fit process outside of the automotive axle industry is possible. For example, the technique may have application in assessing measurement systems and binning strategies for select fit assembly processes. Manufacturing processes where a correlation of in process measurement accuracy to the assembled product final tolerance distribution is a potential application for this methodology.

Suggestions for Further Study

This study selected a single axle shim-selection system to model and validate. A suggestion for further research would be to apply the approach to the correlation of other shim selection systems using the uncertainty framework and the standard uncertainties developed in this study. Evaluating other systems would permit validation and refinement of the uncertainty process.

The results of this research indicate that a significant amount of the uncertainty is attributable to factors not directly measured in the process. The approach did predict the shim selection process capability; this serves as validation that there are significant contributors not in the direct measurement process. An example is bearing torque variation, the uncertainty $u_{STAB} = 0.025$ Nm per bearing. This source of error was based on a Type A statistical study. It is not a measured variable, but it is the most significant contributor to the torque uncertainty. A topic for further research is to quantify and validate these indirect elements. Further research might assess the stability of the bearing torque through the process by detailed in-process measurements on sample parts.

Another topic would be to study the influence of a specific independent variable as a control to compare the response of the system to the uncertainty model. By varying a specific variable in a design of experiments, the response of the system to the variable change could be studied. For example, a significant contributor to backlash uncertainty in this study was the Single Flank Test runout parameter. A Type A method fitting a gamma distribution to this parameter was an assumption in the model but the true effect on process capability for this one parameter was not validated. A study linking the parameter from the SFT through the process could be designed as validation, or as a method to develop a more accurate correlation of this variation to the shim selection process capability.

Conclusion

The results of this study support the conclusion that uncertainty principles in measurement and the uncertainty framework provide a method to correlate independent variables and predict performance of the shim-selection process dependent variables of backlash and torque to rotate. The study applied measurement uncertainty methods defined in literature to model the axle shim-selection measurement process. The uncertainty framework was found to be a viable approach to include known error sources, both those based on statistical analysis, and those determined by non-statistical methods. For the axle shim selection process studied, the difference between the uncertainty model prediction and the actual production system performance, though statistically significant, did not have a practical significance. The study demonstrated the application of an uncertainty model is a method to estimate the effects of varying acceptance limits on the independent process variable measurements. Acceptance limits for an individual in-process shim-selection measurement apparatus can be assessed using this technique. The research indicates that the method is a viable tool for the technology manager to make tradeoffs and optimize an automotive axle shim selection measurement system.

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APPENDIX A: IN-PROCESS MEASUREMENT IMAGES

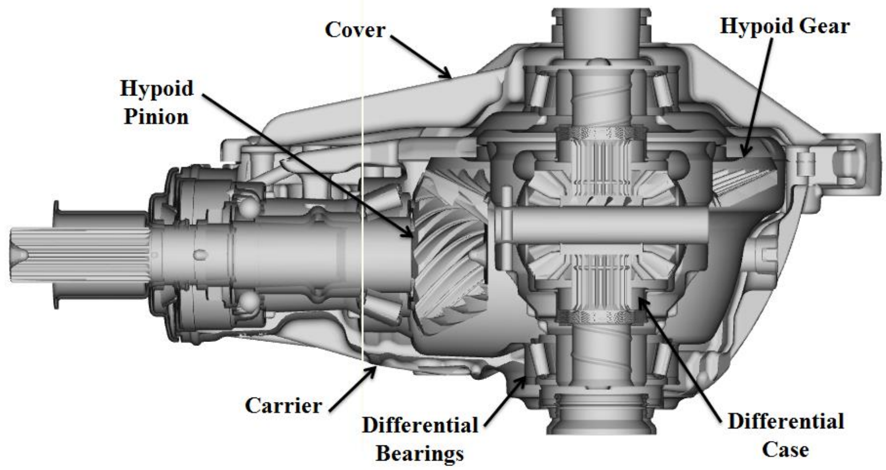

Figure A22. Front axle cross section

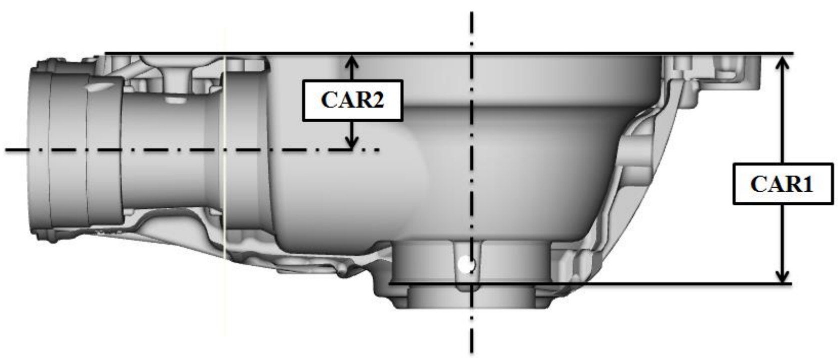

Figure A23. Carrier independent static input measurement variables

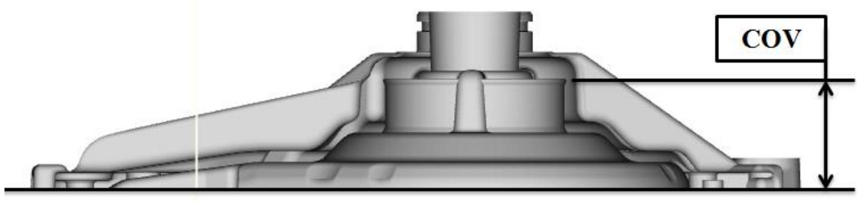

Figure A24. Cover independent static input measurement variable

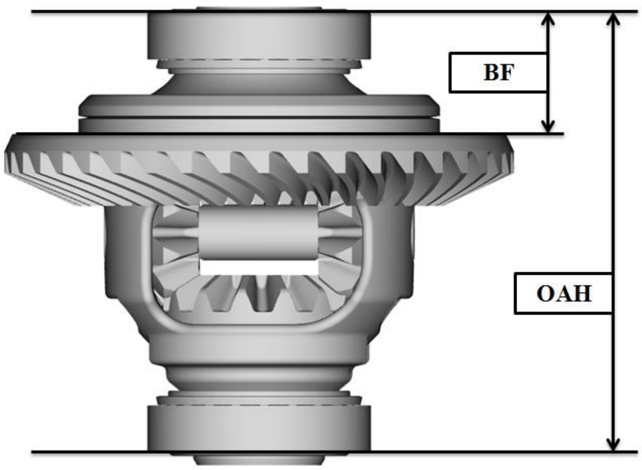

Figure A25. Differential Case dynamic input measurement variables

The dynamic measurement δJ is a measure of the variation of the true gear mounting distance deviation from the theoretical mounting distance.

$$Actual\ Gear\ Mounting\ Distance = G + \delta J$$

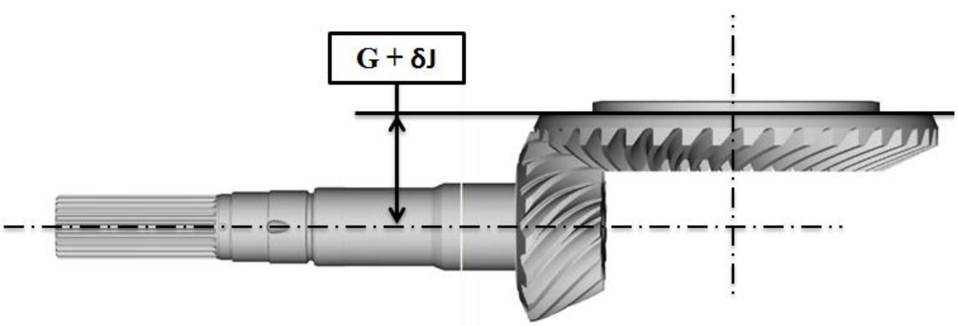

Figure A26. Single Flank Test (SFT) input measurement variable

APPENDIX B: TYPE A REPEATABILITY AND REPRODUCIBILITY STUDIES

CAR1 standard uncertainty from repeatability studies

Independent Variable: CAR1 Uncertainty: u_{EVO}

Method: ISO 22514-7 ANOVA Study Appendix A (ISO, 2012)

Appraiser not significant - $u_{EVO} = \sigma_{Reproducibility}$

Nominal Value 102.XXXX mm (all measurements units are mm)

Table B32

GR&R data collection summary for variable CAR1

	Part									
	1	2	3	4	5	6	7	8	9	10
A 1	0.2178	0.2213	0.2405	0.2300	0.2384	0.2408	0.2276	0.2380	0.2396	0.2393
2	0.2196	0.2224	0.2308	0.2285	0.2414	0.2411	0.2283	0.2388	0.2397	0.2394
3	0.2183	0.2231	0.2254	0.2280	0.2430	0.2412	0.2289	0.2388	0.2398	0.2391
B 1	0.2183	0.2219	0.2405	0.2306	0.2390	0.2407	0.2282	0.2382	0.2396	0.2395
2	0.2201	0.2229	0.2308	0.2306	0.2427	0.2408	0.2288	0.2387	0.2396	0.2392
3	0.2195	0.2230	0.2254	0.2295	0.2437	0.2409	0.2288	0.2389	0.2396	0.2392
C 1	0.2190	0.2224	0.2405	0.2293	0.2408	0.2412	0.2283	0.2382	0.2398	0.2394
2	0.2208	0.2227	0.2252	0.2291	0.2427	0.2405	0.2288	0.2388	0.2397	0.2392
3	0.2200	0.2237	0.2399	0.2289	0.2437	0.2409	0.2287	0.2387	0.2397	0.2391

Table B33

GR&R ANOVA results for variable CAR1

Source	DF	SS	MS	F	p
Part	9	0.00524	0.00058	504	.00
Appraiser	2	$7.2*10^{-6}$	$3.6*10^{-6}$	3.12	.051
Interaction	18	$2.1*10^{-5}$	$1.2*10^{-6}$	0.16	1.00
Reproducibility	60	0.000426	$7.10*10^{-6}$		

$$Estimated\ \sigma^2_{CAR1} = 7.10 * 10^{-6}$$

$$u_{EVO-CAR1} = Estimated\ \sigma_{CAR1} = \sqrt{\sigma^2_{CAR1}} = 0.00266 \text{ mm}$$

CAR2 standard uncertainty from repeatability studies

Independent Variable: CAR2 Uncertainty: u_{EVO}

Method: 22514-7 ANOVA Study Appendix A (ISO, 2012)

Appraiser not significant - $u_{EVO} = \sigma_{Reproducibility}$

Nominal Value 42.XXXX mm (all measurements units are mm)

Table B34

GR&R data collection summary for variable CAR2

	Part									
	1	2	3	4	5	6	7	8	9	10
A 1	0.0084	0.0028	-0.0054	0.0077	0.0021	-0.0065	-0.0088	0.0148	0.0032	-0.0017
2	0.0089	0.0032	-0.0055	0.0076	0.0021	-0.0068	-0.0093	0.0131	0.0026	-0.0018
3	0.0080	0.0024	-0.0058	0.0073	0.0018	-0.0072	-0.0089	0.0136	0.0020	-0.0020
B 1	0.0083	0.0026	-0.0052	0.0074	0.0027	-0.0075	-0.0092	0.0139	0.0023	-0.0027
2	0.0084	0.0031	-0.0060	0.0079	0.0014	-0.0062	-0.0096	0.0142	0.0030	-0.0022
3	0.0094	0.0032	-0.0057	0.0079	0.0010	-0.0071	-0.0096	0.0135	0.0022	-0.0028

C 1	0.0076	0.0023	-0.0054	0.0075	0.0029	-0.0073	-0.0089	0.0138	0.0018	-0.0024
2	0.0083	0.0029	-0.0063	0.0077	0.0018	-0.0069	-0.0089	0.0136	0.0030	-0.0020
3	0.0085	0.0029	-0.0051	0.0081	0.0017	-0.0064	-0.0092	0.0138	0.0025	-0.0019

Table B35

GR&R ANOVA results for variable CAR2

Source	DF	SS	MS	F	p
Part	9	0.00439	0.00049	4076	.00
Appraiser	2	$1.8*10^{-7}$	$9.1*10^{-8}$	0.76	.71
Interaction	18	$2.2*10^{-6}$	$1.2*10^{-7}$	0.58	.90
Reproducibility	60	$1.24*10^{-5}$	$2.06*10^{-7}$		

$$Estimated\ \sigma^2_{CAR2} = 2.06 * 10^{-7}$$

$$u_{EVO-CAR2} = Estimated\ \sigma_{CAR2} = \sqrt{\sigma^2_{CAR2}} = 0.00045 \text{ mm}$$

COV standard uncertainty from repeatability studies

Independent Variable: COV Uncertainty: u_{EVO}

Method: 22514-7 ANOVA Study Appendix A (ISO, 2012)

Appraiser not significant - $u_{EVO} = \sigma_{Reproducibility}$

Nominal Value 55.XXXX mm (all measurements units are mm)

Table B36

GR&R Data collection summary for variable COV

	Part									
	1	2	3	4	5	6	7	8	9	10
A 1	0.5197	0.5197	0.5209	0.5526	0.5526	0.5502	0.5233	0.5404	0.5514	0.5221
2	0.5197	0.5184	0.5221	0.5526	0.5526	0.5514	0.5233	0.5404	0.5502	0.5221
3	0.5197	0.5184	0.5209	0.5526	0.5538	0.5526	0.5233	0.5404	0.5514	0.5221

B 1	0.5197	0.5197	0.5221	0.5526	0.5526	0.5514	0.5233	0.5404	0.5514	0.5221
2	0.5197	0.5184	0.5221	0.5526	0.5538	0.5514	0.5233	0.5392	0.5514	0.5233
3	0.5197	0.5184	0.5209	0.5526	0.5538	0.5526	0.5233	0.5392	0.5514	0.5221
C 1	0.5197	0.5184	0.5209	0.5526	0.5526	0.5514	0.5233	0.5404	0.5514	0.5233
2	0.5197	0.5184	0.5209	0.5526	0.5526	0.5514	0.5233	0.5404	0.5514	0.5233
3	0.5197	0.5184	0.5209	0.5526	0.5538	0.5526	0.5233	0.5416	0.5502	0.5221

Table B37

GR&R ANOVA results for variable COV

Source	DF	SS	MS	F	p
Part	9	0.0197	0.00218	7453	.00
Appraiser	2	$2.22*10^{-7}$	$1.11*10^{-7}$	0.38	.90
Interaction	18	$5.27*10^{-6}$	$2.9*10^{-7}$	1.00	.47
Reproducibility	60	$1.76*10^{-5}$	$2.94*10^{-7}$		

$$Estimated\ \sigma_{COV}^2 = 2.94 * 10^{-7}$$

$$u_{EVO-COV} = Estimated\ \sigma_{CAR2} = \sqrt{\sigma_{CAR2}^2} = 0.00054 \text{ mm}$$

OAH standard uncertainty from repeatability studies

Independent Variable: OAH Uncertainty: u_{EVO}

Method: Study Type 3 Average Range Method (Dietrich & Schulze, 2011)

25 Parts, 2 Measurements each (measurements units are mm)

Table B38

GR&R Data collection summary for variable OAH

Part	Measure 1	Measure 2	Difference
1	134.583	134.581	0.002

2	134.582	134.581	0.001
3	134.513	134.579	0.066
4	134.581	134.583	0.002
5	134.581	134.582	0.001
6	134.582	134.581	0.001
7	134.581	134.582	0.001
8	134.583	134.579	0.004
9	134.582	134.581	0.001
10	134.584	134.582	0.002
11	134.600	134.600	0.000
12	134.601	134.549	0.052
13	134.597	134.602	0.005
14	134.601	134.606	0.005
15	134.600	134.604	0.004
16	134.570	134.570	0.000
17	134.569	134.573	0.004
18	134.572	134.569	0.003
19	134.589	134.568	0.021
20	134.578	134.567	0.011
21	134.563	134.587	0.024
22	134.536	134.534	0.002
23	134.534	134.535	0.001
24	134.535	134.534	0.001
25	134.536	134.535	0.001

Average Range $\bar{\bar{R}} = 0.0086$ mm

Groups>20, subgroup=2: d_2^*=1.12838

$u_{EVO-OAH} = Estimated\ \sigma_{SHIM} = \bar{\bar{R}}/d_2^* = 0.00762$ mm

BF standard uncertainty from repeatability studies

Independent Variable: BF Uncertainty: u_{EVO}

Method: Study Type 3 Average Range Method (Dietrich & Schulze, 2011)

25 Parts, 2 Measurements each (measurements units are mm)

Table B39

GR&R Data collection summary for variable BF

Part	Measure 1	Measure 2	Difference

Part	Measure 1	Measure 2	Difference
1	37.431	37.429	0.002
2	37.432	37.430	0.002
3	37.432	37.429	0.003
4	37.430	37.432	0.002
5	37.431	37.431	0.000
6	37.431	37.430	0.001
7	37.436	37.410	0.026
8	37.436	37.435	0.001
9	37.436	37.437	0.001
10	37.438	37.437	0.001
11	37.479	37.480	0.001
12	37.479	37.478	0.001
13	37.477	37.481	0.004
14	37.481	37.482	0.001
15	37.477	37.482	0.005
16	37.393	37.395	0.002
17	37.395	37.399	0.004
18	37.396	37.395	0.001
19	37.393	37.391	0.002
20	37.402	37.392	0.010
21	37.338	37.338	0.000
22	37.338	37.336	0.002
23	37.336	37.339	0.003
24	37.338	37.337	0.001
25	37.337	37.337	0.000

Average Range $\bar{\bar{R}} = 0.00304$ mm

Groups>20, subgroup=2: $d_2^* = 1.12838$

$u_{EVO-BF} = Estimated\ \sigma_{SHIM} = \bar{\bar{R}}/d_2^* = 0.00269$ mm

BF Runout standard uncertainty from test part inhomogeneity

Independent Variable: BF Runout Uncertainty: u_{OBJ}

Method: Study Type 3 Average Range Method (Dietrich & Schulze, 2011)

25 Parts, 2 Measurements each (measurements units are mm)

Table B40

GR&R data collection summary for BF Runout

Part	Measure 1	Measure 2	Difference
1	0.035	0.034	0.001
2	0.033	0.033	0.000
3	0.034	0.033	0.001
4	0.033	0.033	0.000
5	0.033	0.033	0.000
6	0.033	0.033	0.000
7	0.014	0.013	0.001
8	0.013	0.014	0.001
9	0.013	0.014	0.001
10	0.013	0.013	0.000
11	0.025	0.025	0.000
12	0.025	0.025	0.000
13	0.026	0.026	0.000
14	0.027	0.027	0.000
15	0.026	0.027	0.001
16	0.035	0.034	0.001
17	0.035	0.036	0.001
18	0.035	0.035	0.000
19	0.035	0.035	0.000
20	0.037	0.035	0.002
21	0.014	0.013	0.001
22	0.013	0.013	0.000
23	0.014	0.013	0.001
24	0.012	0.013	0.001
25	0.013	0.013	0.000

Average Range $\bar{\bar{R}} = 0.00052$ mm

Groups>20, subgroup=2: $d_2^* = 1.12838$

$u_{EVO-BF} = Estimated\ \sigma_{SHIM} = \bar{\bar{R}}/d_2^* = 0.00046$ mm

δJ standard uncertainty from repeatability studies

Independent Variable: δJ Uncertainty: $u_{EVO},\ u_{AV}$

Method: 22514-7 ANOVA Study Appendix A (ISO, 2012)

Appraiser is significant - $u_{EVO} = \sqrt{\sigma^2_{Reproducibility}}$, $u_{EVO} = \sqrt{\sigma^2_{Appraiser}}$

10 Parts, 3 Appraisers, 3 Measurements each (measurements units are mm)

Table B41

GR&R data collection summary for variable δJ

	Part									
	1	2	3	4	5	6	7	8	9	10
A 1	-0.144	-0.272	-0.124	0.010	-0.126	-0.125	-0.133	-0.142	-0.109	-0.109
2	-0.144	-0.271	-0.122	0.010	-0.123	-0.122	-0.128	-0.143	-0.103	-0.105
3	-0.141	-0.273	-0.120	0.013	-0.120	-0.118	-0.128	-0.141	-0.101	-0.106
B 1	-0.142	-0.272	-0.119	0.013	-0.119	-0.118	-0.127	-0.139	-0.104	-0.105
2	-0.139	-0.274	-0.120	0.013	-0.120	-0.120	-0.128	-0.142	-0.101	-0.104
3	-0.141	-0.272	-0.121	0.012	-0.116	-0.119	-0.126	-0.138	-0.101	-0.104
C 1	-0.142	-0.27	-0.120	0.012	-0.115	-0.119	-0.127	-0.141	-0.101	-0.105
2	-0.137	-0.271	-0.119	0.012	-0.116	-0.12	-0.127	-0.138	-0.100	-0.103
3	-0.138	-0.267	-0.119	0.015	-0.119	-0.118	-0.126	-0.139	-0.102	-0.104

Table B42

GR&R ANOVA results for variable δJ

Source	DF	SS	MS	F	p
Part	9	0.375	0.0417	19900	.00
Appraiser	2	$1.62*10^{-4}$	$8.08*10^{-5}$	38.6	.03
Interaction	18	$3.77*10^{-5}$	$2.1*10^{-6}$	0.59	.89
Reproducibility	60	$2.14*10^{-4}$	$3.57*10^{-6}$		

$$Estimated\ \sigma^2_{Appraiser} = (MS_A - MS_{Int})/(10 * 3) = 2.62 * 10^{-6}$$

$$u_{EVO-\delta J} = Estimated\ \sigma_{Repro} = \sqrt{3.57 * 10^{-6}} = 0.00189 \text{ mm}$$

$$u_{AV-\delta J} = Estimated\ \sigma_{Appraiser} = \sqrt{2.62 * 10^{-6}} = 0.00162 \text{ mm}$$

δJ standard uncertainty from repeatability on standards

Independent Variable: δJ Uncertainty: u_{EVR}

Method: Repeatability on a Standard (ISO, 2012)

33 Measurements (measurements units are mm)

Table B43

GR&R data collection summary for δJ repeatability on a standard

Trial	Measure	Trial	Measure
1	0.121	18	0.120
2	0.128	19	0.119
3	0.125	20	0.120
4	0.126	21	0.118
5	0.122	22	0.122
6	0.123	23	0.120
7	0.122	24	0.123
8	0.125	25	0.121
9	0.121	26	0.120
10	0.119	27	0.122
11	0.121	28	0.120
12	0.122	29	0.121
13	0.122	30	0.121
14	0.120	31	0.121
15	0.121	32	0.124
16	0.121	33	0.121
17	0.119		

Sum of Squares $= 1.462 * 10^{-4}$

$u_{EVR-\delta J} = Estimated\ \sigma_{EVR} = 0.00214$ mm

Shim Verify standard uncertainty from repeatability on test parts

In-process Measurement: Shim Verify Uncertainty: u_{EVO}

Method: Study Type 3 Average Range Method (Dietrich & Schulze, 2011)

25 Parts, 2 Measurements each (measurements units are mm)

Table B44

GR&R data collection summary for shim verification

Part	Measure 1	Measure 2	Difference
1	2.4491	2.4987	0.0496
2	2.5234	2.5254	0.0020
3	2.5520	2.5530	0.0010
4	2.5757	2.5787	0.0030
5	2.6103	2.6083	0.0020
6	2.6350	2.6340	0.0010
7	2.6468	2.6488	0.0020
8	2.6705	2.6725	0.0020
9	2.6962	2.6952	0.0010
10	2.7268	2.7268	0.0000
11	2.7475	2.7455	0.0020
12	2.7870	2.7801	0.0069
13	2.8078	2.8058	0.0020
14	2.8315	2.8315	0.0000
15	2.8601	2.8631	0.0030
16	2.8779	2.8769	0.0010
17	2.9095	2.9095	0.0000
18	2.9134	2.9164	0.0030
19	2.9608	2.9598	0.0010
20	2.9697	2.9697	0.0000
21	3.0023	2.9993	0.0030
22	3.0260	3.0280	0.0020
23	3.0566	3.0586	0.0020
24	3.0813	3.0833	0.0020
25	3.1069	3.1060	0.0009

Average Range $\bar{\bar{R}} = 0.00370$ mm

Groups>20, subgroup=2: $d_2^* = 1.12838$

$u_{EVO-SHIM} = Estimated\ \sigma_{SHIM} = \bar{\bar{R}}/d_2^* = 0.00328$ mm

PTR standard uncertainty from repeatability on test parts

In-process Measurement: PTR Uncertainty: u_{EVO}

Method: Study Type 3 Average Range Method (Dietrich & Schulze, 2011)

5 Parts, 5 Measurements each (measurements units are Nm)

Table B45

GR&R data collection summary for in-process variable PTR

Trial	Part 1	Part 2	Part 3	Part 4	Part 5
1	4.200	4.294	3.693	3.615	3.613
2	4.205	4.295	3.715	3.642	3.650
3	4.204	4.321	3.720	3.648	3.650
4	4.209	4.341	3.715	3.645	3.681
5	4.223	4.345	3.726	3.660	3.696
Range	0.023	0.051	0.033	0.045	0.083

Average Range $\bar{\bar{R}} = 0.047$ Nm

Groups=5, subgroup=5: $d_2^* = 2.3578$

$u_{EVO-PTR} = Estimated\ \sigma_{PTR} = \bar{\bar{R}}/d_2^* = 0.0199$ Nm

DTRP standard uncertainty from repeatability on test parts

In-process Measurement: DTRP Uncertainty: u_{EVO}

Method: Study Type 3 Average Range Method (Dietrich & Schulze, 2011)

5 Parts, 5 Measurements each (measurements units are Nm)

Table B46

GR&R data collection summary for in-process variable DTRP

Trial	Part 1	Part 2	Part 3	Part 4	Part 5
1	2.780	2.670	2.750	2.720	2.720
2	2.810	2.630	2.720	2.700	2.690
3	2.850	2.640	2.730	2.740	2.700
4	2.800	2.670	2.740	2.730	2.700
5	2.830	2.670	2.790	2.740	2.690
Range	0.070	0.040	0.070	0.040	0.030

Average Range $\bar{\bar{R}} = 0.050$ Nm

Groups=5, subgroup=5: $d_2^* = 2.3578$

$u_{EVO-DTR} = Estimated\ \sigma_{PTR} = \bar{\bar{R}}/d_2^* = 0.0212$ Nm

TTR standard uncertainty from repeatability on test parts

Audit Measurement: TTR (Audit Total Torque to Rotate) Uncertainty: u_{EVO}

Method: Study Type 3 Average Range Method (Dietrich & Schulze, 2011)

10 Parts, 5 Measurements each (measurements units are Nm*)*

Table B47

Repeatability study data collection summary for dependent variable DTR

Trial	Part 1	Part 2	Part 3	Part 4	Part 5	Part 6	Part 7	Part 8	Part 9	Part 10
1	3.113	2.887	2.630	2.930	2.740	2.097	2.777	2.684	2.893	2.634
2	3.181	2.870	2.678	2.933	2.712	2.092	2.763	2.650	2.869	2.616
3	3.139	2.855	2.634	2.932	2.717	2.089	2.776	2.639	2.891	2.610
4	3.138	2.868	2.651	2.953	2.739	2.076	2.776	2.665	2.886	2.627
5	3.139	2.872	2.661	2.961	2.741	2.097	2.792	2.673	2.896	2.612
Range	0.068	0.032	0.048	0.031	0.029	0.021	0.029	0.045	0.027	0.024

Average Range $\bar{\bar{R}} = 0.035$ Nm

Subgroups=10, subgroup Size =5: $d_2^* = 2.3419$

$u_{EVO-TTR} = Estimated\ \sigma_{TTR} = \bar{\bar{R}}/d_2^* = 0.0151$ Nm

TTR standard uncertainty from repeatability on standards

Audit Measurement: TTR (Audit Total Torque to Rotate) Uncertainty: u_{EVR}

Method: Linear regression residuals adapted from ISO Appendix A.1.3 (ISO, 2012)

15 Reference Parts (measurements units are Nm*)*

Table B48

TTR repeatability on a reference standard part study

Part	Reference Standard Value	OP180 Value
1	2.564	2.510
2	2.254	2.285
3	2.429	2.421

Part	Reference Standard Value	OP180 Value
4	2.403	2.400
5	2.448	2.447
6	2.420	2.427
7	2.332	2.344
8	2.188	2.269
9	2.180	2.187
10	2.151	2.141
11	2.549	2.504
12	2.820	2.716
13	2.832	2.678
14	2.773	2.661
15	2.738	2.616

Table B49

TTR Repeatability on standard uncertainty regression model and residuals

Model	Sum of Squares	df	Mean Square	F	Sig.
Regression	.430	1	.430	562.6	.000[b]
Residual	.010	13	.001		
Total	.440	14			

Model	Unstandardized Coefficients B	Std. Error	t	Sig.
(Constant)	.596	.078	7.64	.000
REF	.746	.031	23.7	.000

Residuals Statistics[c]

	Minimum	Maximum	Mean	Std. Deviation	N
Predicted Value	2.201	2.709	2.440	.1752	15
Residual	-.0599	.0405	.0000	.0266	15

a. Dependent Variable: OP180

b. Predictors: (Constant), REF

c. Dependent Variable: OP180

$$u_{EVR-TTR} = \sigma_{Residual} = 0.0266 \text{ Nm}$$

LASH standard uncertainty from repeatability on test parts

Audit Measurement: LASH (Audit Backlash) Uncertainty: u_{EVO}

Method: Study Type 3 Average Range Method (Dietrich & Schulze, 2011)

15 Parts, 5 Measurements each (measurements units are mm*)*

Table B50

Repeatability data collection summary for dependent variable LASH

Trial	Part 1	Part 2	Part 3	Part 4	Part 5
1	0.21	0.23	0.19	0.19	0.22
2	0.20	0.21	0.20	0.19	0.23
3	0.21	0.21	0.22	0.19	0.21
4	0.21	0.21	0.22	0.20	0.20
5	0.22	0.21	0.19	0.20	0.20
Range	0.02	0.02	0.03	0.01	0.03

Average Range $\bar{\bar{R}} = 0.022$ mm

Subgroups=5, subgroup Size =5: $d_2^* = 2.3578$

$u_{EVO-Backlash} = Estimated\ \sigma_{TTR} = \bar{\bar{R}}/d_2^* = 0.0093$ mm

APPENDIX C: CORRELATION COEFFICIENT DATA FOR SHIM DEVIATION

Table C51

Shim reprocess data for correlation coefficient analysis

Case #	PS δ (mm)	GS δ (mm)	TOT δ (mm)	LASH δ (mm)	TTR δ (Nm)	Ratio	CASE TTR (Nm)
1	0.027	0.026	0.052	-0.01	0.098	3.231	0.317
4	0.002	0.049	0.051	-0.04	0.171	3.231	0.553
6	-0.041	0.003	-0.038	-0.04	-0.202	3.231	-0.652
8	-0.004	0.052	0.048	-0.02	0.109	3.231	0.354
10	-0.021	-0.019	-0.039	-0.01	-0.283	3.231	-0.915
12	0.004	0.059	0.063	-0.03	0.190	3.231	0.615
14	-0.050	-0.019	-0.069	-0.01	-0.247	3.231	-0.798
16	-0.031	-0.021	-0.051	-0.01	-0.153	3.727	-0.569
18	-0.026	-0.022	-0.047	0.00	-0.155	3.727	-0.579
20	-0.029	-0.027	-0.055	0.00	-0.146	3.727	-0.543
22	-0.046	-0.032	-0.078	-0.01	-0.223	3.727	-0.831
24	-0.023	-0.025	-0.047	0.00	-0.146	3.727	-0.544
28	-0.016	-0.045	-0.061	0.01	-0.201	3.727	-0.751
30	-0.027	-0.051	-0.078	0.02	-0.279	3.231	-0.903
32	-0.016	-0.046	-0.062	0.02	-0.282	3.231	-0.910
34	-0.033	-0.057	-0.090	0.01	-0.316	3.231	-1.020
36	-0.050	-0.014	-0.064	-0.01	-0.294	3.231	-0.949
38	-0.002	-0.073	-0.075	0.03	-0.256	3.231	-0.827
40	0.076	-0.079	-0.003	0.08	0.014	3.231	0.046
42	-0.020	-0.052	-0.072	0.01	-0.296	3.231	-0.956
44	-0.002	-0.045	-0.047	0.03	-0.136	3.231	-0.440
46	-0.026	-0.039	-0.065	-0.01	-0.147	3.231	-0.474
48	0.104	-0.101	0.003	0.08	-0.021	3.727	-0.077
50	0.003	-0.050	-0.047	0.02	-0.162	3.727	-0.603
52	0.052	-0.085	-0.033	0.07	-0.084	3.727	-0.311
54	0.084	-0.077	0.007	0.07	0.055	3.231	0.179
56	0.111	-0.133	-0.022	0.10	-0.063	3.231	-0.204
58	0.052	-0.083	-0.031	0.06	-0.026	3.231	-0.085

Case #	PS δ (mm)	GS δ (mm)	TOT δ (mm)	LASH δ (mm)	TTR δ (Nm)	Ratio	CASE TTR (Nm)
60	0.102	-0.102	0.000	0.11	0.116	3.231	0.376
62	-0.053	0.053	0.000	-0.06	0.006	3.231	0.018
64	-0.069	0.055	-0.014	-0.07	-0.060	3.231	-0.193
66	-0.027	-0.045	-0.072	0.02	-0.326	3.231	-1.054
68	-0.031	-0.037	-0.068	0.00	-0.198	3.727	-0.739
70	0.003	0.076	0.079	-0.02	0.225	3.727	0.839
73	-0.054	-0.006	-0.060	-0.02	-0.154	3.727	-0.572
75	-0.049	-0.001	-0.050	-0.01	-0.177	3.727	-0.659
77	-0.079	0.024	-0.055	-0.05	-0.236	3.727	-0.880
79	-0.027	-0.055	-0.082	0.03	-0.309	3.727	-1.151
81	0.017	0.037	0.054	0.00	0.259	3.231	0.837
83	-0.050	-0.030	-0.080	-0.02	-0.330	3.727	-1.232
85	0.055	-0.076	-0.021	0.06	-0.012	3.231	-0.039
87	0.075	-0.078	-0.003	0.09	0.048	3.231	0.154
89	-0.083	0.064	-0.019	-0.08	-0.011	3.231	-0.036

Table C52

Backlash (LASH) correlation model summary

					Change Statistics				
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	R Square Change	F Change	df1	df2	Sig. F Change
LASH	.976[a]	.953	.951	.0100	.953	405	2	40	.000

Backlash Unstandardized Coefficients[b]

Model		B	Std. Error	t	Sig.
LASH	(Constant)	-.001	.002	-.710	.482
	PS δ	.474	.039	12.254	.000
	GS δ	-.489	.038	-12.804	.000

a. Predictors: (Constant), GS δ, PS δ

b. Dependent Variable: LASH δ

Table C53

Differential Torque to Rotate (DTR) correlation model summary

					Change Statistics				
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	R Square Change	F Change	df1	df2	Sig. F Change
DTR	.950[a]	.903	.901	.175312	.903	382	1	41	.000

Model		DTR Unstandardized Coefficients[b] B	Std Error	t	Sig.
DTR	(Constant)	-.008	.033	-.251	.803
	TOT δ	11.833	.605	19.55	.000

a. Predictors: (Constant), TOT δ

b. Dependent Variable: DIFF TTR

APPENDIX D: MONTE CARLO SIMULATION SAMPLE

Independent Variable Simulation

Independent Variable True Values are simulated using Excel NORMINV and RAND functions and the product feature upper and lower limits.

$True\ Value = NORMINV(Probability, Mean, Standard\ Deviation)$

Where: $Probability = RAND()$

$Mean = Product\ Feature\ Mean$

$Standard\ Deviation = (Upper\ Limt - Lower\ Limt)/12$

Independent Variable error for each simulation case is simulated using Excel NORMINV and RAND functions and the feature standard uncertainty u_{IV}.

$Error = NORMINV(Probability, Mean, Standard\ Deviation)$

Where: $Probability = RAND() Mean = 0.0$

$Standard\ Deviation = u_{IV}$

Observed Values are simulated by adding the True Value and the measurement error for that case using Excel add function.

$Observed\ Value = True\ Value + Error$

Table D54

Sample Independent Variable True and Observed Values

Variable	True Value	Error	Observed Value
CAR1 (mm)	102.2045	-0.0038	102.2007

Variable	True Value	Error	Observed Value
CAR2 (mm)	41.9977	-0.0037	41.9940
COV (mm)	55.5287	-0.0048	55.5239
DELTAJ (mm)	0.2105	-0.0041	0.2064
OAH (mm)	151.7184	-0.0062	151.7122
BF (mm)	49.1411	-0.0050	49.1361

Pinion Side Shim Simulation

"G" $Constant = 45.50$ mm

$PS_{True\ Value} = (102.2045 - 41.9977 + (45.50 + .2105) - (151.7184 - 49.1411) =$ 3.3400 mm

$PS\ _{Observed} = (102.2007 - 41.9940 + (45.50 + .2064) - (151.7122 - 49.1361) =$ 3.3370 mm

Select Shim Class (0.001-inch increments)

$Shim\ Class = ROUND(3.3370/25.4{,}3) * 25.4 = 3.3274$ mm (0.131 inch)

Shim True value is a random shim from shim class

$PS\ Shim_{CLASS\ ERROR} = NORMINV(RAND(\quad), 0{,}0.00433) = -0.0053$ mm

$PS\ Shim_{True\ Value} = 3.3274 + (-0.0053) = 3.3221$ mm

Pinion Side Shim Error

$PS\ Shim_{Error} = 3.3221 - 3.3400 = -0.0179$ mm

Pinion Side Shim observed value includes the measurement error

$u_{Shim\ Meas} = NORMINV(RAND(\quad), 0{,}0.00367) = -0.00145$ mm

$PS\ Shim_{Observed\ Value} = 3.3221 + (-0.0015) = 3.3206$ mm

Gear Side Shim Simulation

$GS_{True\,Value} = (102.2045 + 55.5287 - 151.7184 - 3.3221) = 2.6927$ mm

$GS_{Calc\,Result} = (102.2007 + 55.5239 - 151.7122 - 3.3206) = 2.6918$ mm

Select Shim Class (0.001-inch increments)

$Shim\ Class = ROUND(2.6918/25.4{,}3) * 25.4 = 2.6924$mm (0.106 inch)

Shim True value is a random shim from shim class

$GS\ Shim_{CLASS\,ERROR} = NORMINV(RAND(\quad), 0{,}0.00433) = -0.0047$ mm

$GS\ Shim_{True\,Value} = 2.6924 + (-0.0047) = 2.6877$ mm

Error in Gear Side Shim Call

$GS\ Shim_{Error} = 2.6877 - 2.6927 = -0.0049$ mm

Shim Selection Process Error

$DTRERR_{SHIM} = 11.83 * (GS_{ERROR}) = 11.83 * (-0.0049) = -0.0585$ Nm

$LASHERR_{SHIM} = -0.489 * GS_{ERROR} + 0.474 * PS_{ERROR} = -0.489 * (-0.0049) +$

$0.474 * (-0.0179) = -0.0061$ mm

Process Measurement Error

Process Variable PTR and DTR True Values are simulated using Excel NORMINV and RAND functions and the process variable mean and limits based on typical bearing variation.

$True\ Value = NORMINV(Probability, Mean, Standard\ Deviation)$

Where: $Probability = RAND()$

$Mean = Mean$

$Standard\ Deviation = 0.14$

Process Variable PTR and DTR error for each simulation case is simulated using Excel NORMINV and RAND functions and the feature standard uncertainty u_{PV}.

$$Error = NORMINV(Probability, Mean, Standard\ Deviation)$$

Where: $Probability = RAND()$

$Mean = 0.0$

$Standard\ Deviation = u_{PV}$

Observed Values for PTR and DTR are simulated by adding the True Value and the measurement error for that case using Excel add function.

$$Observed\ Value = True\ Value + Error$$

Process Variable for Single Flank Test Backlash True Value is simulated using Excel GAMMAINV, RAND, and MEDIAN functions using derived parameters based on SFT data for Gamma distribution and the mean product backlash of 0.18 mm.

$$SFT\ LASH\ Error = GAMMAINV(Probability, Alpha, Beta)$$

Where: $Probability = RAND()$

$Alpha = 8.86$

$Beta = 0.0051027$

$$SFT\ LASH\ True\ Value = 0.18 - MEDIAN(SFT\ LASH\ Error) + SFT\ LASH\ Error$$

Table D55

Process Random True values

Variable	True Value	Error	Observed Value
OP90PTR (Nm)	1.6208	-0.0510	1.5698
OP120DTRP (Nm)	2.2696	-0.0888	2.1807
SFT LASH (mm)	0.1766	-0.0034	0.180
RATIO	42/13	N/A	N/A

Audit True Values

Audit True Values include the Process variable true values and the shim selection process error.

$$DTR_{ASSEMBLY\ TRUE\ VALUE} = 2.2696 + (-0.0585) = 2.2111 \text{ Nm}$$

$$TTR_{ASSEMBLY\ TRUE\ VALUE} = 1.6208 + \left[\frac{2.2111}{(42/13)}\right] = 2.3052 \text{ Nm}$$

$$LASH_{ASSEMBLY\ TRUE\ VALUE} = 0.1766 + (-0.0061) = 0.1705 \text{ Nm}$$

Dependent Variable Observed Values

Dependent Variable error for each simulation case is simulated using Excel NORMINV and RAND functions and the Audit Feature standard uncertainty u_{IV}.

$$Error = NORMINV(Probability, Mean, Standard\ Deviation)$$

Where: $Probability = RAND()$

$Mean = 0.0$

$Standard\ Deviation = u_{DV}$

Table D56

Dependent Variable Observed Values

Variable	True Value	Error	Observed Value
TTR (Nm)	2.3052	-0.0067	2.2985
LASH (mm)	0.1705	0.0132	0.1837

$$DTR_{OBSERVED} = (2.2985 - 1.5698) * (42/13) = 2.354 \text{ Nm}$$

$$DTR_{OBSERVED\ ERROR} = 2.354 - 2.181 = 0.173 \text{ Nm}$$

$$LASH = Round(0.1837, 2) = 0.18 \text{ mm}$$

Table D57

Monte Carlo Output Case #1

Variable	Value	Target	Error
DTRERR (Nm)	0.173	0.000	0.173
LASH (mm)	0.18	0.18	0.00

APPENDIX E: UNCERTAINTY OF ASSEMBLY PINION POSITION “P”

The audit measurement of backlash includes an uncertainty associated with variation in the pinion position, or “P” as shown in Figure 26. This Appendix derives the uncertainty or error of the assembled pinion position based on the process for assembling the pinion. The relationship between hypoid backlash and pinion position “P” error is approximated by 0.24 to 1 ratio, for every 0.010 mm “P” error, backlash will vary 0.0024 mm (Mohsen Koviland PhD, personal communication, November 23, 2016).

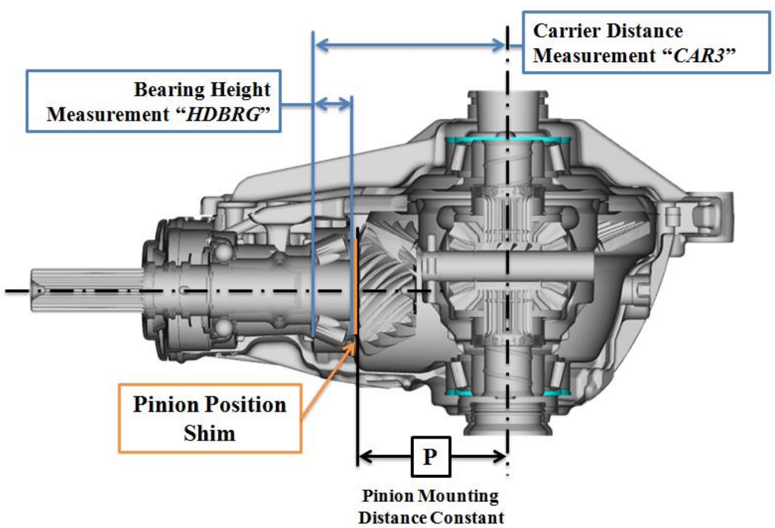

Figure E27. Axle assembly pinion “P” mounting distance

The Pinion Shim Gap Calculation

$Pinion\ Shim\ Gap = CAR3 - HDBRG - P$

Where: *P* is a constant, the design pinion mounting distance (92 mm)

CAR3 is the measured distance gear centerline to bearing seat on the Carrier

HDBRG is the measured bearing height

Example calculation where $CAR3 = 122.230$mm and $HDBRG = 29.450$ mm:

$Pinion\ Shim\ Gap = 122.230 - 29.450 - 92 = 0.780$ mm

Shim classes are in 0.025 mm (0.001 inch) increments, closest shim class is 0.787 mm (0.031 inch).

$Pinion\ Shim = 0.787$ mm

Pinion Position "P" Standard Uncertainty

The selection of the pinion position shim includes two measurements, CAR3 and HDBRG. The uncertainty for these measurements is estimated by Type 3 Study described by Dietrich and Schulze (2011)on test parts to determineu_{EVO}. Standard uncertainties associated with the process contribute to the pinion position variation in the assembly. The selected shim that is installed is nominally at the shim class value analogous to resolution uncertainty per ISO (2012) *Table 2 - Uncertainty from resolution*, the shim selection increment is a rectangular distribution. The uncertainty associated with shim class is, $u_{CLASS} = (1/\sqrt{3}) * (Shim\ Step/2) = Shim\ Step/\sqrt{12} = 0.0254/\sqrt{12} = 0.00733$ mm

The selected shim includes variation within the tolerance range of ±0.0125 mm. A Type B approach applies the part tolerance to determine uncertainty. Given that the tolerance range covers ± 3 standard deviations, the uncertainty is equivalent to the process standard deviation, $u_{SHIM} = 0.025/6 = 0.00423$ mm.

The process for pinion assembly contributes to position variability resulting from the pinion-bearing preload that is created by tightening the pinion nut. The process for setting pinion bearing preload includes measuring the torque to establish a target, and then setting to that target. Three uncertainties are associated with the process, measurement uncertainty of the targeting and setting process, and process capability of the setting process are analyzed using Type A methods.